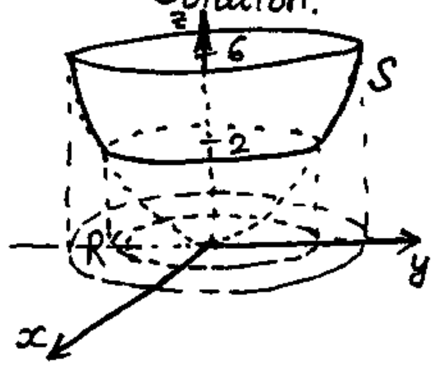
MATH 116-02 QUIZ 14

Surname\ Name:

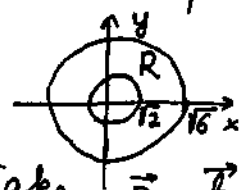
ID:

Problem. Find the area of the band cut from the paraboloid $x^2+y^2-z=0$ by the planes $z=2$ and $z=6$.

Solution:



Surface S is a level surface $f(x,y,z)=x^2+y^2-z=0$ that lies above a plane region R:



Take $\vec{p}=\vec{k}$.

$$\text{Surface Area}(S)=\iint_R \frac{|\nabla f|}{|\nabla f\circ\vec{p}|}\,dA$$

Since $\nabla f=2x\,\vec{i}+2y\,\vec{j}-\vec{k}$, $|\nabla f|=\sqrt{4x^2+4y^2+1}$,

$|\nabla f\circ\vec{p}|=|-1|=1$, then

$$\text{Surface Area}(S)=\iint_R\sqrt{4x^2+4y^2+1}\,dA \underset{\begin{cases}x=r\cos\theta\\ y=r\sin\theta\end{cases}}{=}$$

$$=\int_0^{2\pi}\int_{\sqrt2}^{\sqrt6}\sqrt{1+4r^2}\,r\,dr\,d\theta \underset{\substack{1+4r^2=t\\ 8r\,dr=dt}}{=}$$

$$=\frac18\int_0^{2\pi}\int_9^{25}t^{1/2}\,dt\,d\theta=\frac18\int_0^{2\pi}\frac23 t^{3/2}\Big]_{t=9}^{t=25}d\theta=$$

$$=\frac1{12}(125-27)\int_0^{2\pi}d\theta=\frac{98}{12}\cdot2\pi=\frac{49}{3}\pi.$$