SAMPLE

MATH 116-07 QUIZ 13

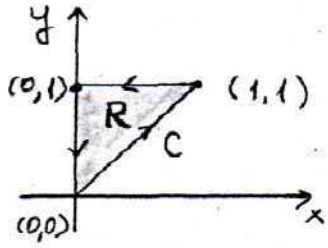
Surname\ Name:

ID:

Problem. Use Green's Theorem to find the counterclockwise circulation and outward flux for the field

$$\mathbf{F} = (y^2 - x^2)\vec{\mathbf{i}} + (x^2 + y^2)\vec{\mathbf{j}}$$

around and across the boundary of the triangle with vertices $(0,0)$, $(0,1)$ and $(1,1)$.



$F = M\vec{i} + N\vec{j}$, where $M = y^2 - x^2$, $N = x^2 + y^2$.

By Green's Theorem,

(a) Outward Flux of F across C $= \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dx\,dy =$

$$= \int_0^1 \int_0^y (-2x + 2y)\, dx\, dy =$$

$$= \int_0^1 \left[-x^2 + 2xy\right]_{x=0}^{x=y} dy = \int_0^1 +y^2\, dy = \frac{y^3}{3}\Big]_0^1 = \frac{1}{3}$$

(b) Counterclockwise circulation of F around C $= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy =$

$$= \int_0^1 \int_0^y (2x - 2y)\, dx\, dy =$$

$$= \int_0^1 \left[x^2 - 2xy\right]_{x=0}^{x=y} dy = \int_0^1 -y^2\, dy = -\frac{y^3}{3}\Big]_0^1 = -\frac{1}{3}$$