SAMPLE

MATH 116-02 QUIZ 11

Surname\ Name:

ID:

Problem 1. Find the work done by force $\mathbf{F} = (3x^2 - 3x)\vec{i} + 3z\vec{j} + \vec{k}$ over the curve C : $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^4\vec{k}$, $0 \le t \le 1$.

Solution:

① $F(x,y,z)\big|_C = (3t^2-3t)\vec{i} + 3t^4\vec{j} + \vec{k}$

② $\frac{d\vec{r}}{dt} = \vec{i} + 2t\vec{j} + 4t^3\vec{k}$

③ $F \circ \frac{d\vec{r}}{dt} = 3t^2 - 3t + 6t^5 + 4t^3$

④ Work done by F over C $= \int_0^1 F \circ \frac{d\vec{r}}{dt}\, dt =$

$= \int_0^1 (3t^2 - 3t + 6t^5 + 4t^3)\, dt = \left(t^3 - \frac{3}{2}t^2 + t^6 + t^4\right)\Big]_{t=0}^{t=1} = \frac{3}{2}.$

Problem 2. Find the flow of the velocity field $\mathbf{F} = (x+y)\vec{i} - (x^2+y^2)\vec{j}$ along the upper half of the circle $x^2 + y^2 = 1$ from $(1,0)$ to $(-1,0)$.

Solution: Curve C : $\begin{cases} x = \cos t \\ y = \sin t \\ 0 \le t \le \pi \end{cases} \longrightarrow \vec{r}(t) = \cos t\,\vec{i} + \sin t\,\vec{j}$

① $F(x,y)\big|_C = (\cos t + \sin t)\vec{i} - \vec{j}$

② $\frac{d\vec{r}}{dt} = -\sin t\,\vec{i} + \cos t\,\vec{j}$

③ $F \circ \frac{d\vec{r}}{dt} = -\cos t \sin t - \sin^2 t - \cos t = -\frac{1}{2}\sin(2t) - \frac{1-\cos 2t}{2} - \cos t$

④ Flow of F along C $= \int_0^\pi F \circ \frac{d\vec{r}}{dt}\, dt = \int_0^\pi \left(-\frac{1}{2}\sin(2t) - \frac{1}{2} + \frac{1}{2}\cos(2t) - \cos t\right) dt$

$= -\frac{\pi}{2}.$