MATH 116: INTERMEDIATE CALCULUS III

Second Midterm Exam
July 5, 2008
14:00 - 16:00

Surname/Name : ______________________________

ID# : ______________________________

Department : ______________________________

Section : ______________________________

- **The exam consists of 5 questions.**
- Please read the questions carefully.
- **Show all your work in legibly written, well-organized mathematical sentences.**
- **What cannot be read will not be read.**
- Calculators and dictionaries are not allowed.
- **Simplify as far as possible unless otherwise stated.**
- Please turn off your cellular phones before the exam starts.

Please do <u>not</u> write below this line.

Q1	Q2	Q3	Q4	Q5	TOTAL
15	25	20	20	20	100

1. (15 points) Calculate

$$\iint_R (3x+1)\,dA$$

where R is the region bounded by $y = x^2$, $y = (x-1)^2$, $y = 0$.

2. a) (15 points) Find the area of the region between the cardioid $r = 1 + \cos\theta$ and the circle $r = \cos\theta$.

2. b) (10 points) Calculate the improper integral

$$\int_0^\infty \int_x^{\sqrt{3}x} e^{-(x^2+y^2)}\, dy\, dx.$$

3. a) (10 points) Use Taylor's formula for $f(x,y) = \displaystyle\int_0^{x+y^2} e^{-t^2}\,dt$ at the origin to find a quadratic approximation of $f(x,y)$ near the origin.

3. b) (10 points) Use the method of Lagrange multipliers to find the volume of the largest (maximum volume) closed rectangular box in the first octant having three faces in the coordinate planes and a vertex on the plane $\frac{x}{3} + \frac{y}{4} + \frac{z}{2} = 1$.

4. Let D be the solid bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the paraboloid $z = \frac{1}{3}(x^2 + y^2)$. Without evaluating the integrals, set up iterated integrals in the following coordinate systems to calculate the volume of D:

a) (5 points) in cartesian coordinates,

b) (5 points) in cylindrical coordinates, and

c) (10 points) in spherical coordinates.

5. (20 points) Evaluate the integral

$$\iint\limits_R \cos\left(\frac{2x+2y}{x-y}\right)\, dA$$

where R is the trapezoidal (yamuksu) region with vertices $(1,0)$, $(2,0)$, $(0,-2)$, and $(0,-1)$.