SAMPLE

MATH 116-02 QUIZ 15

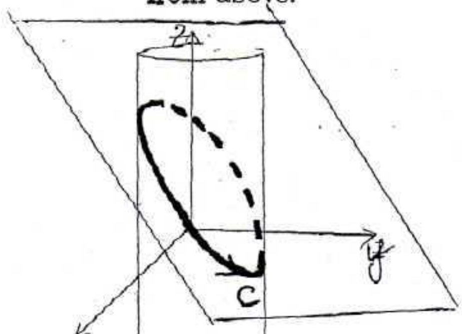
Surname\ Name:

ID:

Problem. Use Stokes' Theorem to evaluate

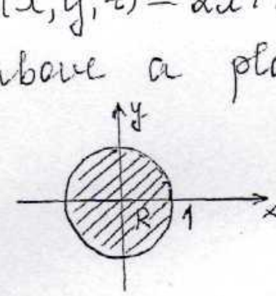
$$\oint_C F \cdot d\mathbf{r},$$

where $\mathbf{F} = -y^3\mathbf{i} + x^3\mathbf{j} - z^3\mathbf{k}$ and C is the curve of intersection of the cylinder $x^2+y^2=1$ and the plane $2x+2y+z=3$ oriented counterclockwise as viewed from above.



Curve C belongs to the plane $2x+2y+z=3$. Therefore, C is the boundary of the surface $S: f(x,y,z) = 2x+2y+z-3=0$ that lies above a plane region

R :



Here, $\vec{p} = \vec{k}$

By Stokes' Theorem,

$$\oint_C F \cdot dr = \iint_S (\nabla \times F)\circ \vec{n}\, d\sigma.$$

We have, 1) $\nabla \times F = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y^3 & x^3 & -z^3 \end{vmatrix} = \vec{i}(0-0) - \vec{j}(0-0) + \vec{k}(3x^2+3y^2)$

2) $\vec{n} = \dfrac{2\vec{i}+2\vec{j}+\vec{k}}{\sqrt{2^2+2^2+1}} = \frac{2}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{1}{3}\vec{k}$

3) $(\nabla \times F)\circ\vec{n} = \vec{k}(3x^2+3y^2)\circ\left(\frac{2}{3}\vec{i}+\frac{2}{3}\vec{j}+\frac{1}{3}\vec{k}\right) = x^2+y^2$

Thus, $\oint_C F\circ dr = \iint_S (\nabla\times F)\circ\vec{n}\, d\sigma = \iint_S (x^2+y^2)\, d\sigma = \iint_R (x^2+y^2)\dfrac{|\nabla f|}{|\nabla f\circ\vec{p}|}\, dA =$

$$= \iint_R (x^2+y^2)\frac{|2\vec{i}+2\vec{j}+\vec{k}|}{|(2\vec{i}+2\vec{j}+\vec{k})\circ\vec{p}|}\, dA = 3\iint_R (x^2+y^2)\, dx\, dy =$$

$$\underset{\text{polar}}{=} 3\int_0^{2\pi}\int_0^1 r^2 \cdot r\, dr\, d\theta = \frac{3\pi}{2}.$$

$x = r\cos\theta$
$y = r\sin\theta$