

MATH 116 INTERMEDIATE CALCULUS III
MIDTERM I EXAM

Date: June 20, 2005, Time: 9:00-11:00

SURNAME/NAME:.....

ID:..... Section.....

- 1** Check that there are 5 questions on your booklet.
- 2** Show all your work. Correct answers without sufficient explanation may not get full credit.

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Problem 1.

(a) Use the ε - δ definition of limit to verify that

$$\lim_{(x,y) \rightarrow (1,-1)} (2x^2 - 4y^2) = -2$$

(b) Find

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin x \sin^3 y}{1 - \cos(x^2 + y^2)}$$

or explain why it doesn't exist.

Problem 2. Let $f(x, y)$ and its first and second order partial derivatives be continuous functions. Suppose

$$f_{xx} + f_{yy} = 0$$

Consider function $w(t, s) = f(t^2 + s^2, t^2 - s^2)$, where $x = t^2 + s^2$ and $y = t^2 - s^2$. Find

$$w_{tt}(1, 1) + w_{ss}(1, 1) \quad ,$$

if it is given that $f_x(2, 0) = 3$.

Problem 3. Find parametric equations for the line tangent to the curve of intersection of the cylinder $4z = 5\sqrt{16 - x^2}$ and the plane $y = 3$ at the point $Q\left(2, 3, \frac{5\sqrt{3}}{2}\right)$.

Problem 4. Find and classify all the critical points of function $f(x, y) = x^2 - xy + y^3 - y$.

Problem 5. Find the shortest distance from the origin to the surface $z^2 = 3 + xy$.