

**MATH 116 INTERMEDIATE CALCULUS III
FINAL EXAM**

Date: July 22, 2005, Time: 9:00-11:00

SURNAME/NAME:.....

ID:..... Section.....

- 1** Check that there are 5 questions on your booklet.
- 2** Show all your work. Correct answers without sufficient explanation may not get full credit.

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Problem 1. Suppose that function $f(x, y, z)$ has continuous second order partial derivatives and $f(x, y, z) = g(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$. Evaluate $f_{xx} + f_{yy} + f_{zz}$ at point $(2, -2, 1)$ given that $g_r(3) = 6$ and $g_{rr}(3) = 1$.

Problem 2. Show that the curve $\vec{r}(t) = \sqrt{t} \vec{i} + \sqrt{t} \vec{j} - \frac{t+3}{4} \vec{k}$ is normal to the surface $x^2 + y^2 - z = 3$ at the intersection point.

Problem 3. Let $F = (3x^2y + z^2)\vec{i} + (x^3 - 2yz)\vec{j} + (2xz - y^2)\vec{k}$ be a vector field.

(a) Show that vector field F is conservative.

(b) Evaluate the counterclockwise circulation $\oint_C F \circ dr$, where C is the graph of function

$$\vec{r}(t) = \sin \frac{t}{2} \vec{i} + \tan \frac{t}{4} \vec{j} + \frac{t}{\pi} \vec{k}, \quad 0 \leq t \leq \pi.$$

Problem 4. Let C be the boundary of the rectangle having vertices at points $(0, 0, 0)$, $(0, 3, 3)$, $(1, 3, 3)$, $(1, 0, 0)$ oriented in the clockwise direction when viewed from high on the z -axis. Find circulation $\oint_C F \circ dr$ of the vector field $F = x^2\vec{i} + 4xy^3\vec{j} + y^2x\vec{k}$ around the curve C .

Problem 5. Let D be the region given by $x^2 + y^2 + z^2 \leq 4a^2$ and $x^2 + y^2 \geq a^2$. Let S be the surface of solid D . Evaluate the flux $\int_S F \circ \vec{n} \, d\sigma$ of the vector field $F = (x + yz)\vec{i} + (y - xz)\vec{j} + (z - e^x \sin y)\vec{k}$ across the surface S .