MATH 116-02 QUIZ 5

Surname\ Name:

ID:

Problem. Use the method of Lagrange multiplier to find the extreme values of $f(x,y,z) = x^2 + y^2 + z^2$ subject to the constraint $z^2 = xy + 4$.

Solution.

We have to maximize/minimize $f(x,y,z) = x^2+y^2+z^2$ subject to the constraint $g(x,y,z) = z^2 - xy - 4 = 0$.

To do so, we have to solve the system

$$\begin{cases} \nabla f = \lambda \nabla g \\ g(x,y,z) = 0 \end{cases} \longrightarrow \begin{cases} 2x\vec{i} + 2y\vec{j} + 2z\vec{k} = +\lambda(-y\vec{i} - x\vec{j} + 2z\vec{k}) \\ z^2 = xy + 4 \end{cases}$$

6 points

$$\begin{cases} 2x = -\lambda y \\ 2y = -\lambda x \\ 2z = 2z\lambda \\ z^2 = xy + 4 \end{cases} \longrightarrow z(1-\lambda) = 0 \longrightarrow \begin{array}{l} \text{Case 1)} \quad z = 0 \\ \text{Case 2)} \quad \lambda = 1 \end{array}$$

In case 1), $z = 0$, $xy = -4$, $x \neq 0, y \neq 0$, $2x = -\lambda y$, $2y = -\lambda x$

$\lambda \neq 0$

$x = -\frac{\lambda y}{2}$, $2y = -\lambda \cdot \left(-\frac{\lambda y}{2}\right)$

$\lambda^2 = 4$

Thus

$\lambda = 2$	or	$\lambda = -2$
$x = -y$		$x = y$
$xy = -4$		$xy = -4$
$(2,-2,0)$		impossible
$(-2,2,0)$		

In case 2), $\lambda = 1$, $2x = -\lambda y \rightarrow 2x = -y$, $2y = -x \longrightarrow 2(-2x) = -x \rightarrow x = 0$, $y = 0$, $z^2 = 4$

$(0,0,2)$, $(0,0,-2)$ 2 points

$f(2,-2,0) = f(-2,2,0) = 8$

$f(0,0,2) = f(0,0,-2) = 4 \leftarrow$ abs min value. 2 points.

There is no abs. max value ($f(t, -\frac{1}{4}t, 0) = t^2 + \frac{1}{16}t^2 \xrightarrow{t \to \infty} +\infty$)