SAMPLE

MATH 116-02 QUIZ 9

Surname\ Name:

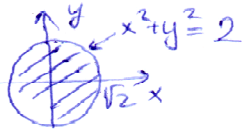
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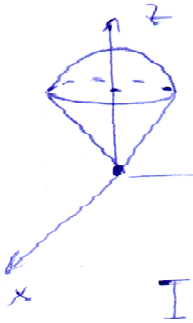
Problem. Convert

$$I = \int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r^2 \, dz \, dr \, d\theta, \qquad r \geq 0,$$

(a) to rectangular coordinates with the order of integration $dz\,dx\,dy$

xy-projection in polar coordinates:

$0 \leq r \leq \sqrt{2}$, $0 \leq \theta \leq 2\pi$ → 



The solid is bounded above by the sphere $x^2+y^2+z^2=4$, below by the cone $z^2=x^2+y^2$

Boundary surfaces

1) $z = r \longrightarrow z^2 = r^2 \Rightarrow z^2 = x^2+y^2$ is a cone

2) $z=\sqrt{4-r^2} \longrightarrow z^2 = 4-x^2-y^2 \longrightarrow x^2+y^2+z^2=4$, $z>0$ is a half sphere

$$I = \int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r \cdot r \, dz \, dr \, d\theta = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2} \, dz \, dy \, dx$$

(b) to spherical coordinates

$$I = \int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r \cdot r \, dz \, dr \, d\theta =$$

$$= \int_0^{2\pi} \int_0^{\pi/4} \int_0^{2} \rho \sin\varphi \; \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{2} \rho^3 \sin^2\varphi \, d\rho \, d\varphi \, d\theta$$

Boundary surfaces: $\rho = 2$ and $\varphi = \frac{\pi}{4}$

$$r = \sqrt{x^2+y^2} = \sqrt{\rho^2\sin^2\varphi\cos^2\theta + \rho^2\sin^2\varphi\sin^2\theta}$$

$$= \rho \sin\theta$$