SAMPLE

MATH 116-02 QUIZ 10

Surname\ Name:

ID:

Problem. Use a suitable substitution to evaluate the integral

$$I = \int\int_R \frac{e^{2x-y}}{x+y} dA,$$

where R is the region bounded by the lines $y = 2x$, $y = 2x + 5$, $y = 2 - x$ and $y = 4 - x$.

Introduce $\begin{cases} u = 2x - y, \\ v = x + y. \end{cases}$

Then $\begin{cases} x = \frac{1}{3}(u+v), \\ y = \frac{1}{3}(-u+2v). \end{cases}$

Therefore, $\frac{\partial(x,y)}{\partial(u,v)} = J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{vmatrix} = \frac{2}{9} + \frac{1}{9} = \frac{1}{3}$

The boundaries in xy-plane	→ The boundaries in uv-plane
$y = 2x$	$u = 0$
$y = 2x + 5$	$u = -5$
$y = 2 - x$	$v = 2$
$y = 4 - x$	$v = 4$

Thus, the region of integration in uv-plane is a rectangle bounded by the lines $u=0$, $u=-5$, $v=2$, $v=4$.

Then,

$$I = \int_2^4 \int_{-5}^0 \frac{e^u}{v} \cdot \left|\frac{1}{3}\right| du\,dv = \frac{1}{3}(e^u)\Big|_{u=-5}^{u=0} \cdot (\ln v)\Big|_{v=2}^{4} =$$

$$= \boxed{\frac{\ln 2}{3}\left(1 - e^{-5}\right)} = \frac{\ln 2}{3} \cdot \frac{e^5 - 1}{e^5}.$$