BİLKENT UNIVERSITY
Department of Mathematics

MATH 240, ORDINARY DIFFERENTIAL EQUATIONS,
Solution of Homework set[1] # 2

U. Muğan

Homework problems from the 8nd, and 9nd Edition of Boyce & DiPrima SECTION 2.2

1. Solve the following D.E. :

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}.$$

Solution: Separable type D.E. and it can be written as

$$(y + e^{y})dy = (x - e^{-x})dx$$

Integrating both sides of the above equation w.r.t. appropriate variables, we get

$$y^2 + 2e^{y} = x^2 + 2e^{-x} + c, \qquad c = \text{constant}.$$

2. Solve the following D.E. :

$$\frac{dy}{dx} = \frac{x^2}{1 + y^2}.$$

Solution: Separable type D.E. Write the D.E. as $(1 + y^2)dy = x^2 dx$ and integrate both sides, then we obtain

$$y + \frac{y}{3} = \frac{x}{3} + c, \qquad c = \text{constant}.$$

3. Solve the following I.V.P. and determine the solution domain:

$$y' = (1 - 2x)y^2, \quad y(0) = -1/6.$$

Solution: Separable type D.E. Writing the D.E. as $y^{-2}dy = (1 - 2x)dx$ and integrating both sides yield

$$-y^{-1} = x - x^2 + c, \qquad c = \text{constant}.$$

Integration constat c can be determined by using the I.C. ($x_0 = 0$ and $y_0 = -1/6$). I.C. gives $c = 6$. Then the explicit form of the solution is

$$y(x) = \frac{1}{x^2 - x - 6} = \frac{1}{(x + 2)(x - 3)}.$$

Therefore, $x = -2$ and $x = 3$ are the points of the discontinuities of the solution. So the solution domain of the I.V.P. is $(-2, 3)$

---

[1] I made every effort to avoid the calculation errors and/or typos while I prepared the solution set. **You are responsible to check all the solutions and correct the errors if there is any.** If you find any errors and/or misprints, please notify me.

4. Solve the following I.V.P. and determine the solution domain:

$$xdx + ye^{-x}dy = 0, \quad y(0) = 1.$$

Solution: Separable type D.E. Writing the D.E. as $-ydy = xe^x dx$ and integrating both sides yield

$$(x-1)e^x = \frac{y^2}{2} + c, \qquad c = \text{constant}.$$

Integration constat c can be determined by using the I.C. ($x_0 = 0$ and $y_0 = 1$). I.C. gives $c = -1/2$. Then the explicit form of the solution is

$$y(x) = \pm\sqrt{2e^x - 2xe^x - 1}$$

We should choose $+$ sign since $y(0) = 1$. The function under the square root becomes negative near $x = -1.7$ and $x = 0.76$.

5. Solve the following I.V.P. and determine the solution domain:

$$y' = xy^3(1+x^2)^{-1/2}, \quad y(0) = 1.$$

Solution: Write the D.E. as $y^{-3}dy = x(1+x^2)^{-1/2}dx$ and integrate both sides yo obtain the solution

$$-\frac{1}{2}y^{-2} = \sqrt{1+x^2} + c, \qquad c = \text{constant}.$$

I.C. implies that $c = -3/2$. Then the explicit form of the solution is

$$y = \frac{1}{\sqrt{3 - 2\sqrt{1+x^2}}}.$$

$+$ sign has been chosen in order to satisfy the I.C. The solution becomes singular when $2\sqrt{1+x^2} = 3$, i.e. $x = \pm\sqrt{5}/2$ are the singularities of the solution. Solution domain is $(-\sqrt{5}/2, \sqrt{5}/2)$.

6. Solve the following I.V.P. and determine the interval in which the solution is valid:

$$y' = \frac{3x^2}{3y^2 - 4}, \quad y(1) = 0.$$

Hint: To find the interval of the definition, look for the points where the integral curve has a vertical tangent.
Solution: This is separable type D.E., Writing the D.E. as $(3y^2 - 4)dy = 3x^2 dx$ and integrating both sides give

$$y^3 - 4y = x^3 + c, \qquad c = \text{constant}.$$

I.C. gives $c = -1$.
Note that from the D.E. $y' \to \infty$ as $y \to \pm 2/\sqrt{3}$, and from the solution of the D.E. these values of y give the respective values of the abscissas $x = -1.276$ and $x = 1.598$.

7. Solve the following D.E.:

$$y' = -\frac{4x+37}{2x+y}.$$

Solution: The given D.E. is of HOMOGENOUS type. Since if we write the D.E. as

$$\frac{dy}{dx} = -2 - \frac{y}{x}\left[2 + \frac{y}{x}\right]^{-1}$$

Hence, if let $v = y/x$, the given D.E. takes the following separable form

$$v' = -\frac{v^2 + 5v + 4}{(2+v)x}$$

If we integrate the above D.E. for v, we obtain

$$(v+4)^2|v+1| = cx^{-3}, \qquad c = \text{constant}.$$

Then the solution of the given D.E. is

$$(y+4x)^2|x+y| = c.$$

8. Solve the following D.E.:

$$y' = -\frac{x^2 - 3y^2}{2xy}.$$

Solution: We can write the given D.E. as

$$y' = \frac{1}{2}\frac{x}{y} - \frac{3}{2}\frac{y}{x}.$$

Therefore the given D.E. is of homogenous type. Letting $v = y/x$ yields

$$xv' = \frac{(1 - 5v^2)}{2v}.$$

which is a separable type D.E. and its solution is

$$-\frac{1}{5}\ln|1 - 5v^2| = \ln|x| + c,, \qquad c = \text{constant}.$$

Then the solution $y(x)$ is

$$5y^2 = x^2 - \frac{c}{|x|^3}.$$