BİLKENT UNIVERSITY
Department of Mathematics

MATH 240, DIFFERENTIAL EQUATIONS, Homework set # 9

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Some selected problems from *Elementary Differential Equations and Boundary Value Problems*, W.E.Boyce, R.C.DiPrima, Fifth Edition

Section 6.1, DEFINITION OF THE LAPLACE TRANSFORM

1) Find the Laplace transform of $f(t) = t^2$.

2) Recall that $\cosh bt = (e^{bt} + e^{-bt})/2$. Find the Laplace transform of $e^{at} \cosh bt$.

3) Recall that $\sin bt = (e^{ibt} - e^{-ibt})/2i$. Find the Laplace transform of $e^{at} \sin bt$.

4) Find the Laplace transformation of $f(t) = t \sin at$.

Section 6.2, SOLUTION OF INITIAL VALUE PROBLEM

5) Find the inverse Laplace transform of the following functions.

a) $F(s) = \frac{4}{(s-1)^3}$.

b) $F(s) = \frac{3s}{s^2-s-6}$.

6) Use the Laplace transform to solve the following I.V.P's.

a) $y'' - y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = -1.$

b) $y'' - 4y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 1.$

c) $y'' - 2y' - 2y = 0, \quad y(0) = 2, \quad y'(0) = 0.$

d) $y'' + w^2 y = \cos 2t, \quad w^2 \neq 4, \quad y(0) = 1, \quad y'(0) = 0.$

e) $y'' - 2y' + 2y = e^{-t}, \quad y(0) = 0, \quad y'(0) = 1.$

f) Find the transformed function $Y(s)$ of the solution of the following D.E.

$$y'' + 4y = \begin{cases} 1, & 0 \leq t < \pi \\ 0, & \pi \leq t < \infty \end{cases} \qquad y(0) = 1, \quad y'(0) = 0.$$

Section 6.3, STEP FUNCTION

7) Find the Laplace transform of

$$f(t) = \begin{cases} 0, & t < 1 \\ t^2 - 2t + 2, & t \geq 1 \end{cases}$$

8) Find the inverse Laplace transform of $F(s) = \frac{e^{-2s}}{s^2+s-2}$.

Section 6.3, DIFFERENTIAL EQUATIONS WITH DISCONTINUOUS FORCING FUNCTIONS

9) Find the solution of the following I.V.P's.

a) $y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 1, \qquad f(t) = \begin{cases} 1, & 0 \leq t < \pi/2 \\ 0, & \pi/2 \leq t < \infty \end{cases}$.

b) $y'' + 4y = \sin t - u_{2\pi}(t)\sin(t - 2\pi), \quad y(0) = 0, \quad y'(0) = 0.$

c) $y'' + y' + \frac{5}{4}y = t - u_{\pi/2}(t - \pi/2), \quad y(0) = 0, \quad y'(0) = 0.$

d) $y'' + y' + \frac{5}{4}y = f(t), \quad y(0) = 0, \quad y'(0) = 1, \quad f(t) = \begin{cases} \sin t, & 0 \le t < \pi \\ 0, & t \ge \pi \end{cases}$