BİLKENT UNIVERSITY
Department of Mathematics

MATH 240, DIFFERENTIAL EQUATIONS, Homework set # 7

U.Muğan April 28, 2004

Some selected problems from *Elementary Differential Equations and Boundary Value Problems*, W.E.Boyce, R.C.DiPrima, Fifth Edition

Section 5.1, SERIES SOLUTIONS OF SECOND ORDER LINEAR DIFF.EQ's

1) Determine the radius of convergence of the following power series.
a) $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n^2}$.
b) $\sum_{n=0}^{\infty} \frac{n}{2^n} x^n$.

2) Determine the Taylor series about the point x_0 for the given function.
a) $f(x) = \sin x, \qquad x_0 = 0$
b) $f(x) = x^2, \qquad x_0 = -1$
c) $f(x) = \ln x, \qquad x_0 = 1$.

3) Determine the a_n so that the equation

$$\sum_{n=1}^{\infty} n a_n x^{n-1} + 2 \sum_{n=0}^{\infty} a_n x^n = 0$$

is satisfied. Try to identify the function represented by the series $\sum_{n=0}^{\infty} a_n x^n$.

Section 5.2, SERIES SOLUTIONS NEAR AN ORDINARY POINT, Part I

4) Solve the following D.E's by meas of the power series about the given point x_0. Find the recursion relation; also find the first four terms in each of the two L.I. solutions. If it is possible find the general term in each solution.
a) $y'' - xy' - y = 0, \qquad x_0 = 0$.
b) $y'' - xy' - y = 0, \qquad x_0 = 1$.
c) $(1 - x)y'' + y = 0, \qquad x_0 = 0$.
d) $2y'' + (x + 1)y' + 3y = 0, \qquad x_0 = 2$.

5) By making the change of variable $x - 1 = t$ and assuming that y is a power series in t, find two L.I. series solutions of

$$y'' + (x - 1)^2 y' + (x^2 - 1)y = 0$$

in powers of $x - 1$. Show that you obtain the same result directly by assuming that y is a Taylor series in powers of $x - 1$ and also expressing the coefficient $x^2 - 1$ in powers of $x - 1$.

6) Consider the I.V.P.

$$y' = \sqrt{1 - y^2}, \qquad y(0) = 0$$

a) Show that $y = \sin x$ is the solution of this initial value problem.
b) Look for the solution of the I.V.P. in the form of a power series about $x = 0$. Find the coefficients up to the term in x^3 in this series.

Section 5.3, SERIES SOLUTIONS NEAR AN ORDINARY POINT, Part II

7) If $y = \phi(x)$ is a solution of

$$y'' + xy' + y = 0; \quad y(0) = 1, \quad y'(0) = 0$$

determine $\phi''(x_0)$, $\phi'''(x_0)$ and $\phi^{(4)}(x_0)$ for the given point x_0.

8) Determine a lower bound for the radius of convergence of series solution about the given points x_0:

$$(x^2 - 2x - 3)y'' + xy' + 4y = 0, \quad x_0 = 4, \quad x_0 = -4, \quad x_0 = 0$$

9) Find the first three terms in each of two L.I. power series solutions in powers of x of

$$y'' + (\sin x)y = 0.$$

10) Find the power series solution in powers of x of the following non-homogenous first order linear D.E.

$$y' - y = x^2.$$

Section 5.4, REGULAR SINGULAR POINTS

11) Find all singular points of the following D.E's and determine whether each one is regular or irregular.

a) $xy'' + (1 - x)y' + xy = 0$.

b) $(1 - x^2)^2 y'' + x(1 - x)y' + (1 + x)y = 0$.

c) $xy'' + e^x y' + (3\cos x)y = 0$.

d) $(\sin x)y'' + xy' + 4y = 0$.

12) Let

$$2xy'' + 3y' + xy = 0.$$

Show that the point $x = 0$ is a regular singular point and find solution of the form $\sum_{n=0}^{\infty} a_n x^n$. Show that there is only one nonzero solution of this form. Thus the general solution can not be found in this manner.

13) Determine whether the point at infinity is an ordinary point, a regular point, or an irregular singular point of the following D.E.

$$x^2 y'' + xy' + (x^2 - \nu^2)y = 0, \qquad \nu = \text{constant}. \quad \text{Bessel equation.}$$

Section 5.5, EULER EQUATION

14)

a) Find the general solution of $x^2 y'' - 5xy' + 9y = 0$ that is valid in any interval not including the singular point.

b) Find the solution of the I.V.P. $4x^2 y'' + xy' - 3y = 0, \quad y(1) = 1, \quad y'(1) = 4$.

c) Find all values of α for which all solutions of $x^2 y'' + \alpha xy' + (5/2)y = 0$ approaches to zero as $x \to 0$.