BİLKENT UNIVERSITY
Department of Mathematics

MATH 240, DIFFERENTIAL EQUATIONS, Homework set # 6

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VARIATION OF PARAMETERS

1) Find a particular solution of the following D.E. by using the variation of parameters. Then check your answer by using the method of undermined coefficients

a) $y'' - y' - 2y = 2e^{-x}$.

b) $y'' + 2y' + y = 3e^{-x}$.

2) Find the general solution of the following D.E's.

a) $y'' + y = \tan x, \quad 0 < x < \pi/2$.

b) $y'' + 4y' + 4y = x^{-2}e^{-2x}, \quad x > 0$.

c) $y'' - 5y' + 6y = R(x), \quad R(x)$ is an arbitaray contionuous function.

3) Verify that the given y_1 and y_2 satisfy the corresponding homogenous equation and find the particular solution.

a) $x^2y'' - x(x+2)y' + (x+2)y = 2x^3 \quad x > 0; \quad y_1 = x, \quad y_2 = xe^x$.

b) $x^2y'' + xy' + (x^2 - 0.25)y = 3x^{3/2}\sin x, \quad x > 0; \quad y_1 = x^{-1/2}\sin x, \quad y_2 = x^{-1/2}\cos x$.

VARIATION OF PARAMETERS, HIGHER ORDER D.E.

4) We can generalize the method of variation of parameters to higher order D.E's. Lets take the following nth order linear non-homogenous D.E.

$$y^{(n)} + p_1(x)y^{(n-1)} + ... + p_{n-1}(x)y' + p_n(x)y = R(x)$$

where $p_i(x), \ i = 1, 2, ..., n$ and $R(x)$ are continuous functions in an open domain (α, β). Its homogenous solution is

$$y_h(x) = c_1y_1(x) + c_2y_2(x) + ... + c_ny_n(x)$$

where $c_1, c_2, ..., c_n$ are constants and $y_1(x), y_2, ..., y_n(x)$ are n L.I. solutions. Lets assume that a particular solution of the form

$$y_p(x) = c_1(x)y_1(x) + c_2(x)y_2(x) + ... + c_n(x)y_n(x).$$

Setting the sum of the terms containing the derivative of $c_i(x)$ in the derivatives of y_p and the D.E. give the following $n \times n$ non-homogenous system of equations for $c_i'(x), \quad i = 1, 2, ..., n$:

$$\begin{aligned} y_1c_1' + y_2c_2' + ... + y_nc_n' &= 0 \\ y_1'c_1' + y_2'c_2' + ... + y_n'c_n' &= 0 \\ y_1''c_1' + y_2''c_2' + ... + y_n''c_n' &= 0 \\ &. \\ &. \\ &. \\ y_1^{(n-1)}c_1' + y_2^{(n-1)}c_2' + ... + y_n^{(n-1)}c_n' &= R(x) \end{aligned} \tag{1}$$

By the Cramer's rule the solution of the above system is

$$c_i'(x) = \frac{R(x)W_i(x)}{W(x)}, \quad 1 = 1, 2, ..., n$$

where $W(x)$ is the Wronskian and W_i is the determinant obtained from W by replacing ith column by the column $(0, 0, ..., 0, 1)$.

By using the above generalization of variation of parameters, find the particular solution of the following D.E's.
a) $y''' + y' = \tan x, \quad 0 < x < \pi/2.$
b) $y''' - y'' + y' - y = R(x),$ $R(x)$ is an arbitaray contionuous function.
c) $y''' - 3y'' + 3y' - y = x^{-2}e^x.$