BİLKENT UNIVERSITY
Department of Mathematics

MATH 240, DIFFERENTIAL EQUATIONS, Homework set # 3

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FUNDAMENTAL SET OF SOLUTIONS

1) Find the Wronskian of the following given pair of functions:
a) $e^{2x}, \quad e^{-3x/2}$.
b) $x, \quad xe^x$.
c) $e^x \sin x, \quad e^x \cos x$.

2) Determine the largest interval in which the given I.V.P. is certain to have a unique twice differentiable solution. Do not find the solution.
a) $(x-1)y'' - 3xy' + 4y = \sin x, \qquad y(-2) = 2, \;\; y'(-2) = 1$.
b) $y'' + (\cos x)y' + 3(\ln|x|)y = 0, \qquad y(2) = 3, \;\; y'(2) = 1$.
c) $(x-2)y'' + y' + (x-2)(\tan x)y = 0, \qquad y(3) = 1, \;\; y'(3) = 2$.

3) Verify that $y_1(x) = 1$ and $y_2(x) = x^{1/2}$ are two L.I. solutions of

$$yy'' + (y')^2 = 0, \qquad x > 0.$$

Then show that $c_1 + c_2x^{1/2}$ is not, in general a solution of the equation. Why not?

4) If Wronskian W of f and g is x^2e^x and $f(x) = x$, find $g(x)$.

5) Verify that the functions y_1 and y_2 are solutions of the following given equations. Do they constitute a fundamental set of solutions.
a) $y'' + 4y = 0, \qquad y_1(x) = \cos 2x, \quad y_2(x) = \sin 2x$.
b) $x^2y'' - x(x+2)y' + (x+2)y = 0, \qquad x > 0, \qquad y_1(x) = x, \qquad y_2(x) = xe^x$.

HOMOGENOUS, CONSTANT COEFFICIENT D.E.

6) Find the general solution of the following D.E.
a) $y'' + 5y = 0$.
b) $y'' - 9y' + 9y = 0$.
c) $2y'' - 3y' + y = 0$.

7) Find the solution of the following I.V.P.
a) $y'' + 4y' + 3y = 0, \qquad y(0) = 2, \quad y'(0) = -1$.
b) $y'' + 8y' - 9y = 0, \qquad y(1) = 1, \quad y'(1) = 0$.

8) Find α so that the solution of the I.V.P.

$$y'' - y' - 2y = 0, \qquad y(0) = \alpha, \quad y'(0) = 2.$$

approaches to zero as $x \to \infty$.

COMPLEX ROOTS OF THE C.E.

9) Find the general solution of the following D.E.
a) $y'' - 2y' + 2y = 0$.
b) $9y'' + 9y' - 4y = 0$.
c) $4y'' + 9y = 0$.

10) Find the general solution of the following I.V.P. and sketch the graph of the solution describe its behavior for increasing x.
a) $y'' + 4y' + 5y = 0, \qquad y(0) = 1, \quad y'(0) = 0$.
b) $y'' + 2y' + 2y = 0, \qquad y(\pi/4) = 2, \quad y'(\pi/4) = -2$.

11) If it is possible transform the following D.E's into one with constant coefficients and then find the general solution.
a) $y'' + xy' + e^{-x^2}y = 0, \qquad -\infty < x < \infty$.
b) $xy'' + (x^2 - 1)y' + x^3 y = 0, \qquad 0 < x < \infty$.

12) Find the transformation $z = u(x)$ to transform the Euler's equation

$$x^2 y'' + \alpha x y' + \beta y = 0, \qquad x > 0$$

where $\alpha,\ \beta$ are real constants, into an equation with constant coefficients.
Use the above result to solve the following D.E. for $x > 0$.
a) $x^2 y'' + xy' + y = 0$.
b) $x^2 y'' - 4xy' - 6y = 0$.