

MATH 240, DIFFERENTIAL EQUATIONS, Homework set # 2

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EXACT EQUATIONS

1) Determine whether or not each of the following equations is exact, if it is exact, find the solution

a) $(3x^2 - 2xy + 2)dx + (6y^2 - x^2 + 3)dy = 0$

b) $\frac{dx}{dy} = -\frac{ax + by}{bx + cy}$

c) $\frac{xdx}{(x^2 + y^2)^{3/2}} + \frac{ydy}{(x^2 + y^2)^{3/2}} = 0$

2) Show that any equation which is separable, that is, of the form $M(x) + N(y)y' = 0$ is also exact.

3) Find an integrating factor and solve the following equations.

a) $(3x^2y + 2xy + y^3)dx + (x^2 + y^2)dy = 0$

b) $y' = e^{2x} + y - 1$

c) $dx + \left(\frac{x}{y} - \sin y\right) dy = 0$

HOMOGENEOUS EQUATIONS

3) Show that the following equations are homogenous and find their solutions.

a) $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$

b) $\frac{dy}{dx} = \frac{4y - 3x}{2x - y}$

c) $\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$

4) Find the general solution of the following differential equation

$$\frac{dy}{dx} = -\frac{4x + 3y + 15}{2x + y + 7}$$

5) Show that if $M(x, y)dx + N(x, y)dy = 0$ is a homogenous equation, then it has

$$\mu(x, y) = \frac{1}{xM(x, y) + yN(x, y)}$$

for an integrating factor to make the equation exact.

MISCELLANEOUS PROBLEMS

5) Classify and solve the followings differential equations or the I.V.P.

a) $xy' + xy = 1 - y, \quad y(1) = 0$

b) $y' = \frac{x}{x^2y + y^3}, \quad \text{Hint: Let } u = x^2$

c) $(3y^2 + 2xy)dx - (2xy + x^2)dy = 0$

d) $(e^x + 1)y' = y - ye^x$

e) $xy' = y + xe^{x/y}$

f) $\frac{dy}{dx} = \frac{y^3}{1 - 2xy^2}, \quad y(0) = 1$

g) $(2y + 1)dx + \left(\frac{x^2 - y}{x}\right)dy = 0$

h) $\frac{dy}{dx} = -\frac{3x^2y + y^2}{2x^3 + 3xy}, \quad y(1) = -2$

k) $(x^2y + xy - y)dx + (x^2y - 2x^2)dy = 0$

6) Show that $y_1 = 1/x$ is a particular solution of the following Riccati equation, and find the more general solution

$$y' = -\frac{1}{x^2} - \frac{y}{x} + y^2.$$