BİLKENT UNIVERSITY
Department of Mathematics

MATH 240, DIFFERENTIAL EQUATIONS, Homework set # 1

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1) Find the general solution of the following differential equations
a) $y' + 2xy = 2xe^{-x^2}$
b) $(1 + x^2)y' + 4xy = (1 + x^2)^{-2}$
c) $xy' + 2y = \sin x, \quad x > 0$

2) Find the solution of the following initial value problems
a) $xy' + (x + 1)y = x, \quad y(\ln 2) = 1$
b) $x^3y' + 4x^2y = e^{-x}, \quad y(-1) = 0$
c) $y' + \dfrac{2}{x}y = \dfrac{\cos x}{x^2}, \quad y(\pi) = 0, \quad x > 0$

3) Find the solution of the following initial value problems. State the interval in which the solution is valid
a) $y' + y = 1/(1 + x^2), \quad y(0) = 0$
b) $(1 - x^2)y' - xy = x(1 - x^2), \quad y(0) = 2$
c) $x(2 + x)y' + 2(1 + x)y = 1 + 3x^2, \quad y(-1) = 1,$

4) Find the general solution of the following differential equation
$y' = (a \cos x + b)y - y^3, \quad a, b = \text{constant}$

5) Solve the following initial value problems. State the interval in which the solution is valid
$y' = \dfrac{3x^2}{3y^2 - 4}, \quad y(1) = 0$
Hint: use the existence and uniqueness theorem.

6) Solve the following initial value problems. State the interval in which the solution is valid
a) $xdx + ye^{-x}dy = 0, \quad y(0) = 1$
b) $\sin 2x \; dx + \cos 3y \; dy = 0, \quad y(\pi/2) = \pi/3$
c) $y' = 2(1 + x)(1 + y^2), \quad y(0) = 0,$

7) For the following equations state the region in xy-plane where the hypotheses of the existence and uniqueness theorem are satisfied.
a) $y' = \dfrac{x - y}{2x + 5y}$
b) $y' = \dfrac{1 + x^2}{3y - y^2}$
c) $y' = (x^2 + y^2)^{3/2}$

8) Solve the following I.V.Problems and determine how the interval in which the solution exists depends on the initial value y_0.
a) $y' = -4x/y, \quad y(0) = y_0$

b) $y' = 2xy^2, \qquad y(0) = y_0$
c) $y' + y^3 = 0, \qquad y(0) = y_0$

9) Consider the I.V.P. $y' = y^{1/3}, \quad y(0) = 0.$
a) Is there a solution that passes through the point $(1, 1)$? If so, find it.
b) Is there a solution that passes through the point $(2, 1)$? If so, find it.
c) Consider all possible solutions of the given I.V.P. Determine the set of values that these solutions have at $x = 2$.