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MATH 225, LINEAR ALGEBRA and DIFFERENTIAL EQUATIONS,
Solution of Homework set[1] # 9

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Homework problems from the 2nd Edition, SECTION 3.5

3(3))[2]

$$A^{-1} = \begin{bmatrix} 6 & -7 \\ -5 & 6 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} 6 & -7 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 33 \\ -28 \end{bmatrix}.$$

5(5))

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix}, \quad \vec{x} = \frac{1}{2} \begin{bmatrix} 4 & -2 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 8 \\ -7 \end{bmatrix}.$$

7(7))

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix}, \quad \vec{x} = \frac{1}{4} \begin{bmatrix} 7 & -9 \\ -5 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 3 \\ -1 \end{bmatrix}.$$

11(11)) Let

$$\big[A|I\big] = \begin{bmatrix} 1 & 5 & 1 & 1 & 0 & 0 \\ 2 & 5 & 0 & 0 & 1 & 0 \\ 2 & 7 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Then do the following row operations R_2-2R_1, R_3-2R_2, R_1+R_3, R_2-2R_3, R_3+3R_2, $(-1)R_3$,...,R_1-2R_2 and obtain

$$\big[A|I\big] = \begin{bmatrix} 1 & 0 & 0 & -5 & -2 & 5 \\ 0 & 1 & 0 & 2 & 1 & -2 \\ 0 & 0 & 1 & -4 & -3 & 5 \end{bmatrix}.$$

Therefore,

$$A^{-1} = \begin{bmatrix} -5 & -2 & 5 \\ 2 & 1 & -2 \\ -4 & -3 & 5 \end{bmatrix}.$$

17(17)) Let

$$\big[A|I\big] = \begin{bmatrix} 1 & -3 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -2 & 2 & 0 & 0 & 1 \end{bmatrix}.$$

Then do the following row operations R_2+R_1, $(-1)R_2$, R_3+2R_2, $(1/4)R_2$, R_3+2R_2, $(1/4)R_3$,..., and obtain

$$\big[A|I\big] = \begin{bmatrix} 1 & 0 & 0 & -(1/2) & -(3/2) & -(3/4) \\ 0 & 1 & 0 & -(1/2) & -(1/2) & -(1/4) \\ 0 & 0 & 1 & -(1/2) & -(1/2) & (1/4) \end{bmatrix}.$$

[1] I made every effort to avoid the calculation errors and/or typos while I prepared the solution set. **You are responsible to check all the solutions and correct the errors if there is any.** If you find any errors and/or misprints, please notify me.

[2] The number in the parenthesis denotes the problem number in the International Edition of the textbook

Therefore,

$$A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & -6 & -3 \\ -2 & -2 & -1 \\ -2 & -2 & 1 \end{bmatrix}.$$

23(23)) Since

$$A^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$$

then

$$X = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 & -5 \\ -1 & -2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 18 & -35 \\ -9 & -23 & 45 \end{bmatrix}.$$

41(41)) This follows immediately from the fact that the ij^{th} element of $\mathbf{AB}$ is the product of the i^{th} row of $\mathbf{A}$ and the j^{th} column of $\mathbf{B}$.

42(42)) Let $\vec{e_i}$ denote the i^{th} row of $\mathbf{I}$. Then , $\vec{e_i}\mathbf{B} = \mathbf{B}_i$, the i^{th} row of $\mathbf{B}$. Hence the result in Problem 41 yields

$$\mathbf{IB} = \begin{bmatrix} \vec{e}_1 \\ \vec{e}_2 \\ . \\ . \\ . \\ \vec{e}_m \end{bmatrix}\mathbf{B} = \begin{bmatrix} \vec{e}_1\mathbf{B} \\ \vec{e}_2\mathbf{B} \\ . \\ . \\ . \\ \vec{e}_m\mathbf{B} \end{bmatrix} = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ . \\ . \\ . \\ \mathbf{B}_m \end{bmatrix} = \mathbf{B}.$$

43(43)) Let $\mathbf{E}_1, \mathbf{E}_2, ..., \mathbf{E}_k$, be the elementary matrices corresponding to the elementary row operations that reduce $\mathbf{A}$ to $\mathbf{B}$. Then Theorem 5 (*If any elementary row operation is performed on the $m \times n$ matrix $\mathbf{A}$, then the result is the product matrix $\mathbf{EA}$, where $\mathbf{E}$ is the elementary matrix obtained by performing the same row operation on the $m \times m$ identity matrix $\mathbf{I}$.*) gives $\mathbf{B} = \mathbf{E}_k\mathbf{E}_{k-1}...\mathbf{E}_2\mathbf{E}_1\mathbf{A} = \mathbf{EA}$, where $\mathbf{E} = \mathbf{E}_k\mathbf{E}_{k-1}...\mathbf{E}_2\mathbf{E}_1$.