BİLKENT UNIVERSITY
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MATH 225, LINEAR ALGEBRA and DIFFERENTIAL EQUATIONS,
Solution of Homework set[1] # 6

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Homework problems from the 2_{nd} Edition, SECTION 3.2

3(3))[2] Since the linear system is already in echelon form, we can find the solution of the system by using the back substitution. If we set $x_3 = t$ then the 2^{nd} eq. gives $x_2 = 2 + 5t$ and the 1^{st} eq. gives $x_1 = 13 + 11t$.

6(6)) Since the linear system is already in echelon form, we can find the solution of the system by using the back substitution. If we set $x_3 = t$ and $x_4 = -4$ from the 3^{rd} eq., then the 2^{nd} eq. gives $x_2 = 11 + 3t$ and the next eq. gives $x_1 = 17 + t$.

9(9)) Since the linear system is already in echelon form, we can find the solution of the system by using the back substitution. Start with $x_4 = 6$ from the 4^{th} eq., the 3^{rd} eq. gives $x_3 = -5$, the 2^{nd} eq. gives $x_2 = 3$, and finally the 1^{st} eq. gives $x_1 = 1$.

13(13)) Let write the augmented coefficient matrix $[\mathbf{A}|\mathbf{b}]$ of the system first.

$$[\mathbf{A}|\mathbf{b}] = \begin{bmatrix} 1 & 3 & 3 & 13 \\ 2 & 5 & 4 & 23 \\ 2 & 7 & 8 & 29 \end{bmatrix}$$

Do the following row operations, $(-2)R_1 + R_2$, $(-2)R_1 + R_3$, $R_2 + R_3$, and $(-1)R_2$ then obtain

$$\mathbf{E} = \begin{bmatrix} 1 & 3 & 3 & 13 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore $x_3 = t$, $x_2 = 3 - 2t$, $x_1 = 4 + 3t$.

NOTE THAT: The echelon matrix is not UNIQUE, so different sequence of elementary row operations may produce a different echelon matrix.

15(15)) Augmented coefficient matrix $[\mathbf{A}|\mathbf{b}]$ of the linear system of equations is

$$[\mathbf{A}|\mathbf{b}] = \begin{bmatrix} 3 & 1 & -3 & -4 \\ 1 & 1 & 1 & 1 \\ 5 & 6 & 8 & 8 \end{bmatrix}$$

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[1] I made every effort to avoid the calculation errors and/or typos while I prepared the solution set. **You are responsible to check all the solutions and correct the errors if there is any.** If you find any errors and/or misprints, please notify me.

[2] The number in the parenthesis denotes the problem number in the International Edition of the textbook

Do the following elementary row operations: Swap R_1 and R_2, $(-3)R_1 + R_2$ and $(-5)R_1 + R_3$, then

$$\mathbf{E} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Last row implies that the system has no solution (inconsistent system).

NOTE THAT: The echelon matrix is not UNIQUE, so different sequence of elementary row operations may produce a different echelon matrix.

17(17)) If you begin with the elementary row operations, $(-2)R_1+R_2$ and $(-3)R_1+R_3$, and continue then you obtain

$$\mathbf{E} = \begin{bmatrix} 1 & -4 & -3 & -3 & 4 \\ 0 & 1 & 0 & -1 & -4 \\ 0 & 0 & 1 & 3 & 5 \end{bmatrix}$$

$x_4 = t$, $x_3 = 5 - 3t$, $x_2 = -4 + t$, $x_1 = 3 - 2t$.

NOTE THAT: The echelon matrix is not UNIQUE, so different sequence of elementary row operations may produce a different echelon matrix.

19(19)) Augmented coefficient matrix of the system is

$$[\mathbf{A}|\mathbf{b}] = \begin{bmatrix} 3 & 1 & 1 & 6 & 14 \\ 1 & -2 & 5 & -5 & -7 \\ 4 & 1 & 2 & 7 & 17 \end{bmatrix}$$

Do the following elementary row operations: Swap R_1 and R_2, $(-3)R_1 + R_2$, $(-4)R_1 + R_3$, then you will obtain

$$\mathbf{E} = \begin{bmatrix} 1 & -2 & 5 & -5 & -7 \\ 0 & 1 & -2 & 3 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore $x_4 = t$, $x_3 = s$, $x_2 = 5 + 2s - 3t$, $x_1 = 3 - s - t$.

NOTE THAT: The echelon matrix is not UNIQUE, so different sequence of elementary row operations may produce a different echelon matrix.

24(24)) Augmented coefficient matrix of the system is

$$[\mathbf{A}|\mathbf{b}] = \begin{bmatrix} 3 & 2 & 0 \\ 6 & k & 0 \end{bmatrix}$$

If we let $(-2)R_1 + R_2$, then we have the following echelon matrix

$$\mathbf{E} = \begin{bmatrix} 3 & 2 & 0 \\ 0 & k-4 & 0 \end{bmatrix}$$

It follows that the given system has only the trivial solution $x_1 = x_2 = 0$ unless $k = 4$, in which case the system has infintely many solutions by $x_1 = -2t/3$, and $x_2 = t$.

25(25)) If we let $(-2)R_1 + R_2$ we obtain the following echelon form

$$\mathbf{E} = \begin{bmatrix} 3 & 2 & 11 \\ 0 & k-4 & 1 \end{bmatrix}$$

of the augmented coefficient matrix. So, system has unique solution if $k \neq 4$, but no solution if $k = 4$.

29(29)) In each of parts (a)-(c), we start with a typical 2×2 matrix $\mathbf{A}$ and carry out two row successive operations as indicated, observing that we wind up with the original matrix $\mathbf{A}$.
(a)

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \to \alpha R_2 \to \begin{bmatrix} a & b \\ \alpha c & \alpha d \end{bmatrix} \to (1/\alpha)R_2 \to \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \mathbf{A}.$$

(b)

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \to \text{Swap}(R_1, R_2) \to \begin{bmatrix} c & d \\ a & b \end{bmatrix} \to \text{Swap}(R_1, R_2) \to \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \mathbf{A}.$$

(c)

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \to \alpha R_1 + R_2 \to \begin{bmatrix} a & b \\ \alpha a + c & \alpha b + d \end{bmatrix} \to (-\alpha)R_1 + R_2 \to \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \mathbf{A}.$$

Since we therefore can "reverse" any single elementary row operation, it follows that we can reverse any finite sequence of such operations on at a time so part (d) follows.