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MATH 225, LINEAR ALGEBRA and DIFFERENTIAL EQUATIONS, Solution of Homework set[1] # 5

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Homework problems from the 2nd Edition, SECTION 3.1

3(3))[2] Substruction of 3/2 times the first equation from the second eq. gives

$$y/2 = 3/2$$

So, $y = 3$ and from the first equation (back substitution) $x = -4$.

4(4)) Substruction of 6/5 times the first equation from the second eq. gives

$$11y/5 = 44/5$$

So, $y = 4$ and from the first equation $x = 5$.

6(6)) Substruction of 3/2 times the first equation from the second eq. gives $0 = 1$. So, the system is inconsistent and hence no solution exists.

7(7)) Note that second eq. is -2 times the first equation. So we can choose $y = t$ arbitrarily. Then the first eq. gives $x = -10 + 4t$.

11(11)) First swap the first and second equations. Then substraction of twice the new first equation from the new second eq. gives $y - z = 7$, and substraction 3 times the new first eq. from the third eq. gives $-2y + 3z = -18$. The solution of these last equations gives $y = 3,\ z = -4$. Finally substitution of these values of y and z in the new first eq. gives $x = 1$.

13(13)) Substruct the second eq. from the first eq. and get the new first eq. $x + 2y + 3z = 0$. Then substraction of 2 times the new first eq. from the second eq. gives $3y - 2z = 0$, and substraction of 2 times the new first eq. from the third eq. gives $2y - z = 0$. Solution of these two equations gives $y = 0,\ z = 0$. Substitution of these in the new first eq. gives $x = 0$. Therefore the system has trivial solution.

15(15)) If we subtract the first eq. from the second eq., we have $-4y + z = -2$. Subtract 3 times the first eq. from the second eq. and divide the obtained eq. by 2, then obtain $-4y + z = -5/2$. These later two equations are inconsistent, so the system is inconsistent. Therefore no solution.

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[1] I made every effort to avoid the calculation errors and/or typos while I prepared the solution set. **You are responsible to check all the solutions and correct the errors if there is any.** If you find any errors and/or misprints, please notify me.

[2] The number in the parenthesis denotes the problem number in the International Edition of the textbook

19(19)) Substruct of 2 times the first eq. from the second eq. and get $3y - 6z = 9$. Then substraction of the first eq. from the third eq. gives $y - 2z = 3$. Note that last two equations are scalar multiples of each other, so we can choose $z = t$ arbitrarily. Therefore, $y = 3+2t$ and then $x = 8+3t$.

21(21)) Substruct of 3 times the first eq. from the second eq. and get $3y - 6z = 9$. Then substraction of 4 times the first eq. from the third eq. gives $-3y + 9z = -6$. Note that last two equations are scalar multiples of $y - 3z = 2$, so we can choose $z = t$ arbitrarily. Therefore, $y = 2 + 3t$ and then $x = 3 - 2t$.

32(32))
a. The graph of each of these linear equations in x, y, and z is a plane in xyz-space. If these two planes are parallel that is, do not intersect then the equations have no solution. Otherwise, they intersect in a straight line, and each of the infinitely many different points (x, y, z) on this line provides a solution of the system.
b. When $d_1 = d_2 = 0$, we have a homogenous system. Origin of the coordinate system is a point on both planes. Therefore both planes intersect in a straight line, and each of the infinitely many different points (x, y, z) on this line provides a solution of the system.

34(34))
a. If the three planes are parallel and distinct, then they have no common point of intersection, so the system has no solution.
b. If the three planes coincide, then each of the infinitely many different points (x, y, z) of this common plane provides a solution of the system.
c. If two of the planes coincide and are parallel to the third plane, then the three planes have no common point of intersection, so the system has no solution.
d. If two of the planes intersect in a line that is parallel to the third plane, then the three planes have no common point of intersection, so the system has no solution.
e. If two of the planes intersect in a line that lies in the third plane, then each of the infinitely many different points (x, y, z) of this line provides a solution of the system.
f. If two of the planes intersect in a line that intersects the third plane in a single point, then this point (x, y, z) provides the unique solution of the system.