

BILKENT UNIVERSITY
Department of Mathematics

MATH 225, LINEAR ALGEBRA and DIFFERENTIAL EQUATIONS
Homework set # 16

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FUNDAMENTAL SET OF SOLUTIONS

1) Find the Wronskian of the following given pair of functions:

- a) e^{2x} , $e^{-3x/2}$.
- b) x , xe^x .
- c) $e^x \sin x$, $e^x \cos x$.

2) Determine the largest interval in which the given I.V.P. is certain to have a unique twice differentiable solution. Do not find the solution.

- a) $(x - 1)y'' - 3xy' + 4y = \sin x$, $y(-2) = 2$, $y'(-2) = 1$.
- b) $y'' + (\cos x)y' + 3(\ln |x|)y = 0$, $y(2) = 3$, $y'(2) = 1$.
- c) $(x - 2)y'' + y' + (x - 2)(\tan x)y = 0$, $y(3) = 1$, $y'(3) = 2$.

3) Verify that $y_1(x) = 1$ and $y_2(x) = x^{1/2}$ are two L.I. solutions of

$$yy'' + (y')^2 = 0, \quad x > 0.$$

Then show that $c_1 + c_2x^{1/2}$ is not, in general a solution of the equation. Why not?

4) If Wronskian W of f and g is x^2e^x and $f(x) = x$, find $g(x)$.

5) Verify that the functions y_1 and y_2 are solutions of the following given equations. Do they constitute a fundamental set of solutions.

- a) $y'' + 4y = 0$, $y_1(x) = \cos 2x$, $y_2(x) = \sin 2x$.
- b) $x^2y'' - x(x + 2)y' + (x + 2)y = 0$, $x > 0$, $y_1(x) = x$, $y_2(x) = xe^x$.

HOMOGENOUS, CONSTANT COEFFICIENT D.E.

6) Find the general solution of the following D.E.

- a) $y'' + 5y' = 0$.
- b) $y'' - 9y' + 9y = 0$.
- c) $2y'' - 3y' + y = 0$.

7) Find the solution of the following I.V.P.

- a) $y'' + 4y' + 3y = 0$, $y(0) = 2$, $y'(0) = -1$.
- b) $y'' + 8y' - 9y = 0$, $y(1) = 1$, $y'(1) = 0$.

8) Find α so that the solution of the I.V.P.

$$y'' - y' - 2y = 0, \quad y(0) = \alpha, \quad y'(0) = 2.$$

approaches to zero as $x \rightarrow \infty$.

COMPLEX ROOTS OF THE C.E.

9) Find the general solution of the following D.E.

a) $y'' - 2y' + 2y = 0$.

b) $9y'' + 9y' - 4y = 0$.

c) $4y'' + 9y = 0$.

10) Find the general solution of the following I.V.P. and sketch the graph of the solution describe its behavior for increasing x .

a) $y'' + 4y' + 5y = 0$, $y(0) = 1$, $y'(0) = 0$.

b) $y'' + 2y' + 2y = 0$, $y(\pi/4) = 2$, $y'(\pi/4) = -2$.

11) If it is possible transform the following D.E's into one with constant coefficients and then find the general solution.

a) $y'' + xy' + e^{-x^2}y = 0$, $-\infty < x < \infty$.

b) $xy'' + (x^2 - 1)y' + x^3y = 0$, $0 < x < \infty$.

VARIABLE COEFFICIENT D.E., EULER'S EQUATION

12) Find the transformation $z = u(x)$ to transform the Euler's equation

$$x^2y'' + \alpha xy' + \beta y = 0, \quad x > 0$$

where α, β are real constants, into an equation with constant coefficients.

Use the above result to solve the following D.E. for $x > 0$.

a) $x^2y'' + xy' + y = 0$.

b) $x^2y'' - 4xy' - 6y = 0$.