BİLKENT UNIVERSITY
Department of Mathematics

MATH 225, LINEAR ALGEBRA and DIFFERENTIAL EQUATIONS,
Solution of Homework set[1] # 1

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Homework problems from the 2nd Edition, SECTION 1.3

12 (22))[2] Since, $f(x,y)=x\ln y$, and

$$\frac{\partial f}{\partial y}=\frac{x}{y}$$

are both continues in a neighborhood of $(1,1)$, by the theorem of existence and uniqueness the solution exists and unique in some neighborhood of $(1,1)$.

14 (24)) Similar to previous problem, $f(x,y)=y^{1/3}$ is continuous in a neighborhood of $(0,0)$, but $\partial f/\partial y=(1/3)y^{-2/3}$ is not, so the theorem guarantees existence but not uniqueness in some neighborhood of $x=0$.

15 (25)) $f(x,y)=(x-y)^{1/2}$ is not continuous at $(2,2)$ because it is not even defined if $y>x$. Hence the theorem guarantees neither existence nor uniqueness in any neighborhood of the point $x=2$.

17 (27)) Since

$$f(x,y)=\frac{x-1}{y}$$

and

$$\frac{\partial f}{\partial y}=-\frac{x-1}{y^2}$$

are both continuous near the point $(0,1)$, so the theorem guarantees both existence and uniqueness of a solution in some neighborhood of $x=0$.

18 (28)) Neither $f(x,y)=(x-1)/y$ nor $\partial f/\partial y=-(x-1)/y^2$ is continuous near $(1,0)$, so the existence-uniqueness theorem guarantees nothing.

20 (30)) Both $f(x,y)=x^2-y^2$ and $\partial f/\partial y=-2y$ are continues near $(0,1)$. So by the theorem of existence and uniqueness the solution exists and unique in some neighborhood of $x=0$.

27) Just take the derivatives of the given $y(x)$ for $x\le c$ and for $x>c$ and substitute in to D.E., you will see that the D.E. is identically satisfied. So the given $y(x)$ solve the D.E. If $b<0$ then the initial value problem $y'=2\sqrt{y}$, $y(0)=b$ has no solution, because the square root of a negative number would be involved. If $b>0$ we get a unique solution curve through $(0,b)$ defined for all x by following a parabola down (and leftward) to the $x-$axis and then following the $x-$axis to the left.

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[1] I made every effort to avoid the calculation errors and/or typos while I prepared the solution set. **You are responsible to check all the solutions and correct the errors if there is any.** If you find any errors and/or misprints, please notify me.

[2] The number in the parenthesis denotes the problem number in the International Edition

But starting at $(0,0)$ we can follow the positive $x-$axis to the point $(0,c)$ and then branching off on the parabola $y=(x-c)^2$. This gives infinitely many different solutions if $b=0$.

28) I.V.P. $xy'=y,\ y(a)=b$ has a unique solution off the y-axis where $a\neq 0$; infinitely many solutions through the origin where $a=b=0$; no solution if $a=0$ but $b\neq 0$ (so the point (a,b) lies on the positive or negative $y-$axis).

29) If we sketch the graph of the solution, we see that we can start at the point (a,b) and follow a branch of a cubic up or down to the $x-$axis, then follow the $x-$axis an arbitrary distance before branching off (down or up) on another cubic. This gives infinitely many solutions of the initial value problem $y'=3y^{2/3},\ y(a)=b$ that are defined for all x. However, if $b\neq 0$ there is only a single cubic $y=(x-c)^3$ passing through (a,b), so the solution is unique near $x=a$.

30) The function $y=\cos(x-c)$ satisfies the given D.E. $y'=-\sqrt{1-y^2}$ on the interval $c<x<c+\pi$ where $\sin(x-c)>0$, so it follows that

$$-\sqrt{1-y^2}=-\sqrt{1-\cos^2(x-c)}=-\sqrt{\sin^2(x-c)}=-\sin(x-c)=y'.$$

If $|b|>1$ then the initial value problem

$$y'=-\sqrt{1-y^2},\qquad y(a)=b,$$

has no solution because the square root of a negative number would be involved. If $|b|<1$ then there is only one curve of the form $y=\cos(x-c)$ through the point(a,b) this give a unique solution. But if $b=\pm 1$ then we can combine a left ray of the line $y=+1$, a cosine curve from the line $y=+1$ to the line $y=-1$, and then a right ray of the line $y=-1$. If we sketch the graph of the solution, we see that this gives infinitely many solutions (defined for all x) through any point of the form $(a,\pm 1)$.

31) The function $y=\sin(x-c)$ satisfies the D.E. on the interval $c-(\pi/2)<x<c+(\pi/2)$ where $\cos(x-c)>0$, so it follows that

$$\sqrt{1-y^2}=\sqrt{1-\sin^2(x-c)}=\sqrt{\cos^2(x-c)}=-\sin(x-c)=y'.$$

If $|b|>1$ then the initial value problem

$$y'=\sqrt{1-y^2},\qquad y(a)=b$$

has no solution because the square root of a negative number would be involved. If $|b|<1$ then there is only one curve of the form $y=\sin(x-c)$ through the point(a,b); this give a unique solution. But if $|b|=\pm 1$ then we can combine a left ray of the line $y=-1$, a sine curve from the line $y=-1$ to the line $y=+1$, and then a right ray of the line $y=+1$. If we sketch the solution curve, we see that this gives infinitely many solutions (defined for all x) through any point of the form $(a,\pm 1)$.