Coupled optical microcavities in one-dimensional photonic bandgap structures

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Abstract
We present a detailed theoretical and experimental study of the evanescent coupled optical microcavity modes in one-dimensional photonic bandgap structures. The coupled-cavity samples are fabricated by depositing alternating hydrogenated amorphous silicon nitride and silicon oxide layers. Splitting of the eigenmodes and formation of a defect band due to interaction between the neighbouring localized cavity modes are experimentally observed. Corresponding field patterns and the transmission spectra are obtained by using transfer matrix method (TMM) simulations. A theoretical model based on the classical wave analogue of the tight-binding (TB) picture is developed and applied to these structures. Experimental results are in good agreement with the predictions of the TB approximation and the TMM simulations.

Keywords: Photonic bandgap, microcavity, localization, tight-binding approximation

1. Introduction

In recent years, there has been much interest in the physics and applications of one-dimensional spatially periodic, quasiperiodic and random photonic bandgap (PBG) structures [1, 2]. Localization of light in disordered and quasiperiodic photonic systems has been widely studied [3, 4]. Superluminal tunnelling through one-dimensional PBG materials has also inspired great interest [5–7]. Properties of metallo-dielectric one-dimensional PBG structures have been investigated [8–11]. By using one-dimensional PBG structures, many interesting applications have been reported, such as second-harmonic generation [12], pulse compression [13], optical limiting and switching [14,15], filters [16, 17], and photonic band edge lasers [18]. Moreover, the modification of spontaneous emission from atoms placed in one-dimensional PBG structures has been demonstrated [19–22].

By introducing a defect into the PBG structures, it is possible to obtain highly localized cavity modes inside the photonic stop band, which is analogous to the impurity states inside the semiconductor bandgap [23]. Since high-quality cavities have a crucial role in most of the photonic-crystal-based applications, it is very important to investigate the properties of cavities in these structures. In recent years, coupled microcavities (CMCs) have been investigated [24], and used in various applications [22, 25, 26]. These structures consist of two or more planar Fabry–Perot microcavities which are coupled to each other.

In the present work, we give a detailed experimental and theoretical analysis of the CMCs in one-dimensional photonic crystals. These structures are composed of amorphous silicon nitride and silicon oxide multilayers with coupled Fabry–Perot microcavities. This paper is organized as follows: in section 2, we first develop the classical wave analogue of the tight binding (TB) approximation in photonic crystals. Then we derive expressions for the eigenmode splitting, dispersion relation and group velocity corresponding to the coupled microcavity structures. The measured resonant frequencies of the coupled microcavity modes will be compared with the transfer matrix method (TMM) simulations and the TB results in section 3.
2. Tight-binding description of localized modes in photonic crystals

Similarity between the Schrödinger equation and Maxwell’s equations allows us to use many important tools that were originally developed for electronic systems. For example, it is well known that the TB method has proven to be very useful to study the electronic properties of solids [27]. Recently, the classical wave analogue of the TB picture [28] has successfully been applied to photonic structures [29–34]. By using direct implications of the TB picture, a novel propagation mechanism for photons along localized coupled cavity modes in photonic crystals was proposed [30, 33] and demonstrated [34, 35]. In these structures, photons can hop from one tightly confined mode to the neighboring one due to the weak interaction between them (figure 1). We experimentally observed the mode splitting [34], waveguiding [35], heavy photons [36], and EM-beam splitting [37] in coupled-cavity systems in three-dimensional photonic crystals at microwave frequencies. In addition, we reported the strong enhancement of spontaneous emission throughout the cavity band of the one-dimensional coupled optical microcavity structures [22].

In this section, by using the TB approach [33, 34], we first obtain eigenvalues and eigenvectors corresponding to two and three CMCs (figure 1 shows the schematics of a one-dimensional coupled microcavity structure). Then, the expressions for dispersion relation and group velocity are derived. We first consider an individual localized mode \( E_{\Omega}(r) \) of a single cavity that satisfies a simplified version of the Maxwell equations

\[
\nabla \times (\nabla \times E_{\Omega}(r)) = \epsilon_0(r)(\Omega/c)^2 E_{\Omega}(r),
\]

where \( \epsilon_0(r) \) is the dielectric constant of the single cavity and \( \Omega \) is the frequency corresponding to the cavity mode. Here it is assumed that \( E_{\Omega}(r) \) is real, nondegenerate and orthonormal; i.e., \( \int dr \, \epsilon_0(r) E_{\Omega}(r) \cdot E_{\Omega}(r) = 1 \).

In the case of two weakly interacting cavities, we can write the corresponding eigenmode as a superposition of the individual evanescent cavity modes as \( E_{\Omega}(r) = A E_{\Omega}(r) + B E_{\Omega}(r - \Lambda \hat{x}) \). The eigenmode \( E_{\Omega}(r) \) also satisfies equation (1) where \( \epsilon_0(r) \) is replaced with the dielectric constant of the coupled system \( \epsilon(r) = \epsilon(r - \Lambda \hat{x}) \), and \( \Omega \) replaced with eigenfrequency \( \omega \) of the coupled cavity mode. Inserting \( E_{\Omega}(r) \) into equation (1), and multiplying both sides from the left first by \( E_{\Omega}(r) \) and then by \( E_{\Omega}(r - \Lambda \hat{x}) \) and spatially integrating the resulting equations, we obtain the following eigenmodes and eigenfrequencies:

\[
E_{\omega_{1,2}}(r) = \frac{E_{\Omega}(r) \pm E_{\Omega}(r - \Lambda \hat{x})}{\sqrt{2}},
\]

\[
\omega_{1,2} = \Omega \sqrt{\frac{(1 \pm \beta)}{(1 \pm \alpha)}},
\]

where the TB parameters are given by \( \alpha = \int dr \, \epsilon(r) E_{\Omega}(r) \cdot E_{\Omega}(r) \) and \( \beta = \int dr \, \epsilon_0(r - \Lambda \hat{x}) E_{\Omega}(r) \cdot E_{\Omega}(r - \Lambda \hat{x}) \).

Similarly, the eigenmodes of three coupled cavities can be obtained as

\[
E_{\Omega}(r) = \frac{E_{\Omega}(r) \pm \sqrt{2} E_{\Omega}(r - 2\Lambda \hat{x})}{2},
\]

\[
E_{\omega_{1,3}}(r) = \frac{E_{\Omega}(r) \pm \sqrt{2} E_{\Omega}(r - \Lambda \hat{x}) + E_{\Omega}(r - 2\Lambda \hat{x})}{2}.
\]

The corresponding eigenvalues are given by

\[
\Gamma_2 = \Omega,
\]

\[
\Gamma_{1,3} = \Omega \sqrt{\frac{1 \pm \sqrt{2} \beta}{1 \pm \sqrt{2} \alpha}}.
\]

To derive equations (4)–(6), we ignore the second-nearest-neighbour coupling between the cavity modes. When we consider an array of cavities in which each cavity interacts weakly with neighbouring cavities, the eigenmode can be written as a superposition of the individual cavity modes

\[
E(r) = E_0 \sum_n e^{-i\kappa n\Lambda} E_{\Omega}(r - n\Lambda \hat{x}),
\]

where the summation over \( n \) includes all the cavities. The dispersion relation for this structure can be obtained from equations (1) and (7) keeping only the nearest-neighbour coupling terms

\[
\omega(k) = \Omega[1 + \kappa \cos(k\Lambda)].
\]

Here \( \kappa = \beta - \alpha \) is a TB parameter which can be obtained from the splitting of the eigenmodes of two coupled cavities. After finding \( \Omega \), \( \omega_1 \) and \( \omega_2 \) from measurements or simulations, one can determine \( \beta \) and \( \alpha \) by using equation (3).

The group velocity of photons along the localized coupled-cavity modes is given by

\[
v_g(k) = \nabla \omega(k)/k = -\kappa \Lambda \Omega \sin(k\Lambda).
\]

Notice that all physical quantities including dispersion relation and group velocity depend on only a single TB parameter \( \kappa \), and this parameter can be controlled by changing the properties of cavities and the intercavity distance.

3. Coupled optical microcavities: experiment versus theory

Five different samples are used to investigate the coupled optical microcavity structures. The first sample is a 15-pair
distributed Bragg reflector (DBR). The other samples contain one, two, three and seven cavities, respectively. Each pair in these structures consists of \( \lambda_0/4 \) thick alternating silicon nitride (Si\(_3\)N\(_4\)) and silicon oxide (SiO\(_2\)) layers. The corresponding thicknesses and refractive indices are determined by using a Rudolph AutoEL III ellipsometer as \( d_{\text{SiO}_2} = 97.0 \) nm, \( d_{\text{Si}_3\text{N}_4} = 70.3 \) nm and \( \eta_{\text{SiO}_2} = 1.48, \eta_{\text{Si}_3\text{N}_4} = 2.10 \). The cavity layers are silicon oxide with 194 nm (deposition direction). The localized cavity field, \(|E_0(r)|^2\), exhibits an oscillatory behaviour, and most of the field is concentrated around the cavity region.

For two coupled cavities, the transmission characteristics as a function of wavelength are measured and calculated. As shown in figure 2(c), we observe that the resonance mode is split into two distinct symmetric and antisymmetric modes. The measured values of the resonance wavelengths are 599.3 nm (\( \omega_1 = 500.6 \) THz) and 561.1 nm (\( \omega_2 = 534.7 \) THz), which are very close to the calculated results, i.e. 601.6 and 560.6 nm. At this point, we can determine the TB parameters, \( \alpha \) and \( \beta \), by inserting measured or calculated values of \( \omega_1 \) and \( \omega_2 \) into equation (3). This procedure leads to a TB parameter \( \kappa = -0.066 \) when we use experimentally determined values of \( \omega_1 \) and \( \omega_2 \). Similarly, the corresponding resonant frequencies obtained by the TMM simulations lead to \( \kappa = -0.07 \), which is very close to the experimentally obtained value.

The calculated field patterns corresponding to these modes are plotted in figures 3(b) and (c). It is observed that although both field patterns show two peaks around the cavity regions they also exhibit different properties between the cavities. The field intensity corresponding to the lower-frequency mode (antisymmetric) has a node between the cavities. This result is along our expectations, as the localized photon modes should overlap when two isolated cavities are brought together. Due to this interaction, the doubly degenerate eigenmode splits into two distinct modes as we described in the previous section (see equation (2))[39]. These modes are reminiscent of the bonding and antibonding states in solid state physics. For example, in the diatomic molecules, the interaction between the two atoms produces a splitting of the degenerate atomic levels into bonding and antibonding orbitals [40]. Recently, the splitting...
Coupled optical microcavities in one-dimensional photonic bandgap structures

Figure 4. Calculated field patterns of three coupled cavities for the resonant wavelengths (a) $\lambda = \lambda_1$, (b) $\lambda = \lambda_2$, and (c) $\lambda = \lambda_3$. These modes can be constructed by the superposition of the individual cavity mode which is localized at each cavity site.

of photon modes has also been observed in photonic molecules which were fabricated by coupling pairs of micrometre-sized semiconductor cavities [41].

When we brought three cavities together, the single-cavity mode $\Omega_0$ split into three different eigenmodes. In this case, the corresponding transmission spectra are measured and calculated. As shown in figure 2(d), there is good agreement between measured and calculated transmission characteristics of the three coupled cavities. Figure 4 exhibits corresponding field patterns of three coupled cavities. The first and third modes, figures 4(a) and (c), are linear combinations of three individual localized cavity modes with appropriate coefficients, which are given in equation (5). The second mode, figure 4(b), is obtained by combining only the first and third cavity modes, which corresponds to equation (4). As shown in figure 4, there is a exact correspondence between the calculated field patterns by using the TMM and predictions of the TB analysis.

We also compare the measured and the calculated resonant wavelengths of three CMCs with the TB approximation results, which are determined by inserting the parameters $\alpha$ and $\beta$ (these parameters can be obtained from either measured or calculated values of $\omega_1$ and $\omega_2$ by using equation (3)) into the equation (6). Table 1 gives a comparison between the measured and calculated resonance wavelengths of three coupled cavities with the TB predictions. These parameters can be determined from the experiments and TMM simulations. The measured and calculated (by using the TMM code) values of the resonance frequencies coincide well with the TB approximation’s predictions. This excellent agreement shows that the classical wave analogue of TB formalism is a useful tool to investigate PBG structures.

When more cavities are brought together, due to coupling between localized cavity modes, we expect that a cavity band is formed within the PBG. Figure 5(a) displays the measured and simulated transmission spectra of CMC structures which contain seven cavities. The cavity band extends from 540 to 626 nm. Nearly 100% transmission is achieved throughout the cavity band. The calculated transmission spectra agree well with our measurements. It is important to note that since the cavity band edges are very sharp compared to

<table>
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<th>TMM $\alpha$</th>
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$^a$ Simulated results are used to determine $\alpha$ and $\beta$ by using equation (3).

$^b$ Experimental results are used to determine $\alpha$ and $\beta$ by using equation (3).
the cavity sites. These modes are observed. The TB parameters are extracted from measurements after finding the TB parameters from measurements. Figure 6 shows the calculated dispersion relation of the CMC structure using equation (8) with $\kappa = -0.07$. The group velocity is plotted according to equation (9) as a function of wavevector $k$. The group velocity is one order of magnitude smaller than the speed of light at the band centre ($kA = \pi /2$), and vanishes at the band edges ($kA = 0$ and $\pi$).

We can obtain the dispersion relation of the CMC structure after finding the TB parameters from measurements or simulations. Figure 6(a) shows the calculated dispersion relation $\omega(k)$ as a function of wavevector $k$ by using equation (8) with $\kappa = -0.07$. We also plotted the normalized group velocity (equation (9)) corresponding to the CMC. As shown in figure 6(b), the group velocity has its maximum value, nearly one-tenth of the speed of light, at the coupled-cavity band centre, and vanishes at the band edges. The coupled-microcavity structures can efficiently be used in certain applications such as dispersion compensators and photonic switches. Moreover, the spontaneous emission rate and the efficiency of nonlinear processes can be enhanced in coupled-microcavity systems due to very low group velocity.

4. Conclusions

In summary, the transmission properties of the coupled-microcavity structures in one-dimensional PBG materials have been investigated. The structures are fabricated by using hydrogenated amorphous silicon nitride and silicon oxide multilayers. The splitting of eigenmodes due to interaction between the localized electromagnetic cavity modes is observed. The TB parameters are extracted from measurements and the TMM simulation results.

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Coupled optical microcavities in one-dimensional photonic bandgap structures


