Problem 1:

a. Given the following probability density:

$$P_P(t) = \exp(-t/\tau)$$

Find the normalization.

$$A \int_0^\infty e^{-t/\tau}\, dt$$
$$= A\, e^{-t/\tau}(-\tau)\Big|_0^\infty = A\tau = 1$$
$$\boxed{A = 1/\tau}$$

b. The distribution $P_P(t)$ is called the Poisson distribution, and it is often used to describe decay processes. Calculate the probability that a nucleus whose decay time is τ will decay before τ.

$$\frac{1}{\tau}\int_0^\tau e^{-t/\tau} dt = \frac{1}{\tau} e^{-t/\tau}(-\tau)\Big|_0^\tau = \boxed{1-e}$$

c. Calculate the mean and the standard deviation for the Poisson distribution.

mean: $\frac{1}{\tau}\int_0^\infty e^{-t/\tau}\, t\, dt = \frac{1}{\cancel{\tau}}\left[-\cancel{\tau}\, t e^{-t/\tau}\Big|_0^\infty + \cancel{\tau}\int_0^\infty e^{-t/\tau} dt\right]$ $\to 0$

$u = t \quad dv = e^{-t/\tau} dt$

$du = dt \quad v = (-\tau)e^{-t/\tau}$

$$= \int_0^\infty e^{-t/\tau} dt = \boxed{\tau}$$

standard deviation: $\sqrt{\langle t^2\rangle - \langle t\rangle^2} = ?$

$$F(\tau) = \int_0^\infty e^{-t/\tau} dt = \tau$$

$$-\frac{dF(\tau)}{d(1/\tau)} = \int_0^\infty e^{-t/\tau}\, t\, dt = \tau^2$$

$$\frac{d^2F(\tau)}{d(1/\tau)^2} = \int_0^\infty e^{-t/\tau}\, t^2 dt = 2\tau^3$$

$$\langle t^2\rangle = \frac{2\tau^3}{\tau} = 2\tau^2 \qquad \langle t\rangle = \tau$$

$$\boxed{\sqrt{\langle t^2\rangle - \langle t\rangle^2} = \tau}$$

Problem 2:

Consider a system with L sites with spins which can be up or down on each site. Given that there are N of the L sites are up, calculate the number of configurations $\Omega(N)$. (Hint: this is similar to the *Fermionic* situation in problem 1.1 of the text book.)

The entropy is defined as $S=k\ ln\ \Omega(N)$. Give an expression for the entropy of this spin system for the case of N upspins on L sites. (Hint: use the Stirling approximation.)

$$\boxed{\Omega(N) = \frac{L!}{N!\,(L-N)!}}$$

entropy:

$$\boxed{S = k \ln \Omega \cong k\left[L \ln L - N \ln N - (L-N)\ln(L-N)\right]}$$

Stirling $\ln N! = N \ln N - N$

Problem 3:

The probability that a car passes by a certain point on a certain road in time dt is $P(dt) = dt/\tau$. Calculate the probability that in time T exactly two cars pass by.

use $dt \sim \Delta t$ and then take continuous limit $T = N\Delta t$ $N \to \infty$ $\Delta t \to 0$

P_2 - probability of two cars passing by

$$P_2 = \left(\frac{\Delta t}{\tau}\right)^2 \left(1 - \frac{\Delta t}{\tau}\right)^{N-2} \frac{N!}{2!\,(N-2)!}$$

$$= \left(\frac{\Delta t}{\tau}\right)^2 \left(e^{-\frac{\Delta t}{\tau}}\right)^{N-2} \frac{N!}{2!\,(N-2)!}$$

$$= \left(\frac{\Delta t}{\tau}\right)^2 e^{-\frac{N\Delta t}{\tau}} e^{\frac{2\Delta t}{\tau}} \frac{N!}{2!\,(N-2)!}$$

$$= \left(\frac{N\Delta t}{\tau}\right)^2 e^{-\frac{N\Delta t}{\tau}} e^{\frac{2\Delta t}{\tau}} \frac{N!}{2!\,N^2(N-2)!}$$

$N \to \infty$ $\Delta t \to 0$ $N\Delta t = T$

$$P_2 = \frac{1}{2!}\left(\frac{T}{\tau}\right)^2 e^{-T/\tau} \qquad \underbrace{\frac{N(N-1)}{N^2}}_{\to 1}$$

$$\boxed{P_2 = \frac{1}{2!}\left(\frac{T}{\tau}\right)^2 e^{-T/\tau}}$$