SOLUTIONS

PHYS552: Statistical Mechanics (Hour exam)

Name:
ID number:
Date: 24/2/2014
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Problem 1:

1. Suppose that an experimentalist reports that a particular material has the following properties:

$$\left(\frac{\partial S}{\partial T}\right)_\mu \geq 0, \qquad \left(\frac{\partial N}{\partial \mu}\right)_T \leq 0.$$

Are these results compatible with stability?

2. Consider a system whose energy depends only on the temperature. In this case the volume is given by *V(P/T)* (this was shown in problem 1.6.1 part 2.). Use the Massieu function to derive stability criteria.

1.) $dE = TdS + \mu dN$ E convex in S, N

$\Omega = E - TS \mp \mu N$

$d\Omega = -SdT - Nd\mu$ Ω concave in T, μ

$\Rightarrow \left.\frac{\partial S}{\partial T}\right)_\mu$ and $\left.\frac{\partial N}{\partial \mu}\right)_T$ must have same sign $\Rightarrow$ NOT COMPATIBLE

2.) Massieu: $\Rightarrow S(N, V, E)$ – concave in E, V

$$\Phi\left(\frac{1}{T}, \frac{P}{T}, N\right) = S - \frac{E}{T} - \frac{PV}{T}$$

$$d\Phi = -E\,d\left(\frac{1}{T}\right) - V\,d\left(\frac{P}{T}\right)$$

Φ – convex in $\frac{1}{T}, \frac{P}{T}$

$$\Phi = \Phi_1\left(\frac{1}{T}\right) + \Phi_2\left(\frac{P}{T}\right)$$

$$E\left(\frac{1}{T}\right) = -\frac{\partial \Phi_1(1/T)}{\partial(1/T)} \qquad V\left(\frac{P}{T}\right) = \frac{-\partial \Phi_2(P/T)}{\partial(P/T)}$$

$$\frac{\partial E(1/T)}{\partial(1/T)} \leq 0 \qquad \frac{\partial V(P/T)}{\partial(P/T)} \leq 0$$

CON'T on worksheet

Problem 2:

Consider two identical copper blocks, *A* and *B*, of mass *m*, specific heat *C* and respective temperatures T_A and T_B with $T_A \le T_B$. We assume that *C* does not depend on the temperature and that the volumes of the blocks are constant. We put *A* and *B* in contact while maintaining the ensemble thermally insulated.

1. What is the change in the internal energy of each of the blocks? What is the total energy change? What is the final temperature?
2. What is the entropy change of each of the blocks? What is the total entropy change?
3. We construct a heat engine which uses the two blocks as heat sources. What is the maximum work that can be obtained? What is the final temperature of the two blocks in this case?

1.) at Equilibrium T is ~~the~~ same for A and B

$\Delta E_A = -\Delta E_B$ → total energy change is zero

$$C(T_f - T_A) = -C(T_B - T_f)$$

$$\boxed{T_f = \frac{T_A + T_B}{2}}$$

2.) Block A: $\Delta S_A = \int_{T_A}^{T_f} \frac{C}{T}\, dT = C \ln T_f/T_A$

$$= C \ln \frac{T_A + T_B}{2T_A}$$

Block B: $\Delta S_B = C \ln \frac{T_A + T_B}{2T_B}$

total entropy change: $\Delta S = \Delta S_A + \Delta S_B = C \ln \frac{(T_A+T_B)^2}{4T_A T_B}$

3.) $\eta = 1 - \frac{T_A}{T_B} = \frac{W}{\Delta Q_B} = \frac{\Delta Q_B - \Delta Q_A}{\Delta Q_B}$

$$\frac{\cancel{C}(T_f - T_B) - \cancel{C}(T_A - T_f)}{\cancel{C}(T_f - T_B)} = 1 - \frac{T_A}{T_B} = \frac{2T_f - T_A - T_B}{T_f - T_B}$$

$\rightarrow (T_B - T_A)(T_f - T_B) = 2T_f T_B - T_A T_B - T_B^2$

Problem 3:

Consider a container of liquid in an adiabatically isolated cylindrical container of radius R. The container is equipped with a propeller at the end of a rod which is placed along the axis of the cylinder. The propeller can be dipped into and lifted out of the liquid. The propeller is run by a motor connected to a battery. (See figure.)

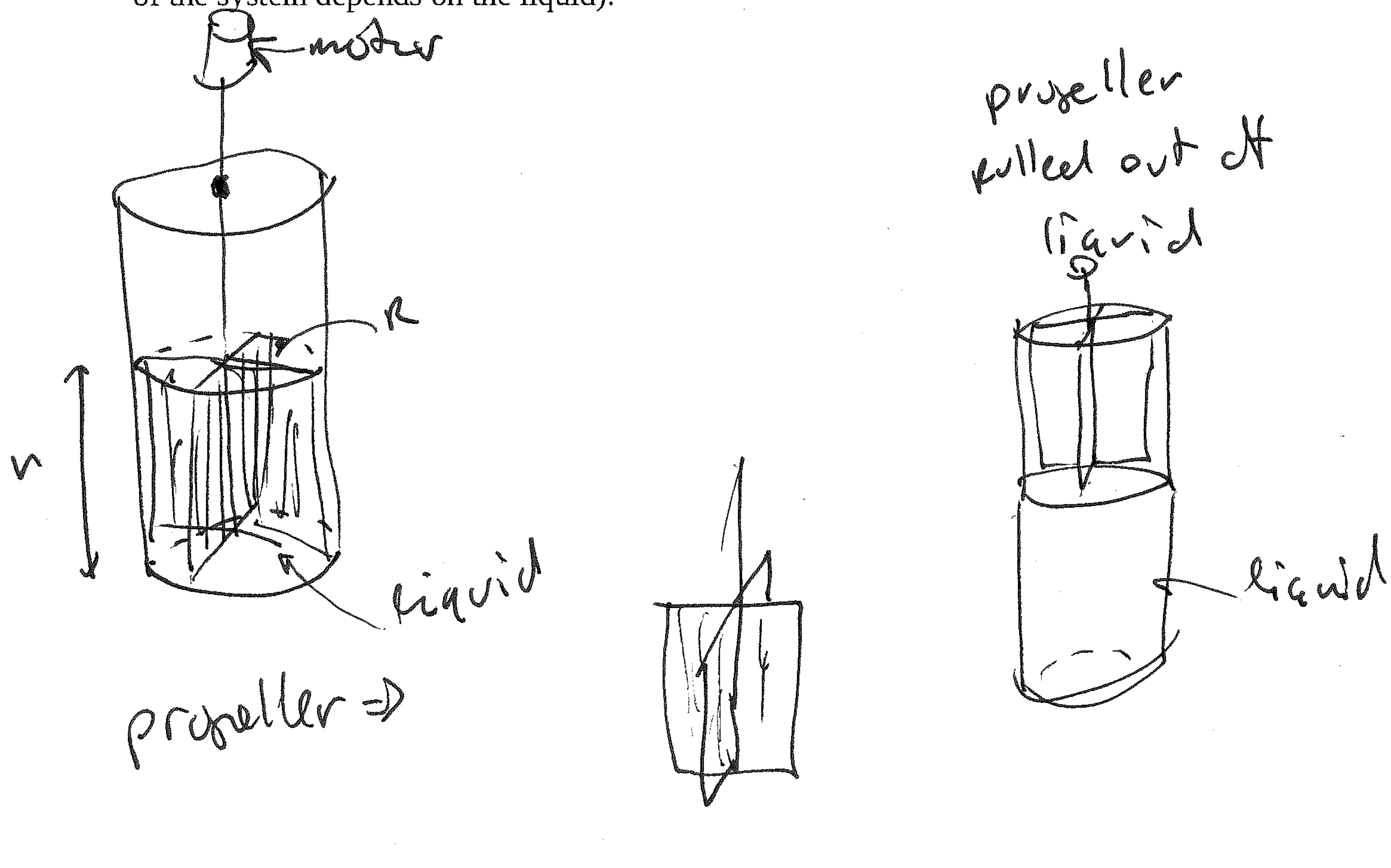
Suppose that the container is filled with a liquid whose specific heat is a temperature independent constant C. The height of the liquid is h, its density is ρ, its temperature is T. The liquid is not superfluid (i.e. the particles can dissipate energy by collisions with the wall of the container) . At time $t=0$, the propeller is dipped inside the liquid and the motor is turned on. The motor rotates the liquid such that its angular velocity reaches a value of ω by time τ. At that time the propeller is pulled out of the liquid and the motor is turned off.

Calculate

1. the work done on the system.
2. the heat supplied to the system.
3. the temperature of the liquid.

at time τ and at time ∞.

Assumet that the effect of dissipation via collisions with the adiabatic walls is negligible before time τ. Assume also that the adiabatic walls are of negligible size (i.e. the energy of the system depends on the liquid).



Problem 2 (con't)

$$\cancel{T_B} T_f - T_A T_f - \cancel{T_B^2} + 2 T_A T_B = T_f T_B +$$

Worksheet:

$$\boxed{T_f = \frac{2 T_A T_B}{T_A + T_B}}$$

max. work: $C(T_B - T_f) - C(T_f - T_A)$

$$C(T_A + T_B) - 2CT_f$$

$$C(T_A + T_B) - \frac{4C\, T_A T_B}{(T_A + T_B)}$$

$$\boxed{\delta W = \frac{C\,(T_A - T_B)^2}{(T_A + T_B)}}$$

Problem 3.

Worksheet:

at time τ

work done is $\frac{I\omega^2}{2}$ $\quad I = h\rho\int_0^R r^3\,dr\,2\pi$

$I = \frac{h\rho R^4}{4} 2\pi$

$I = \frac{MR^2}{2}$ $\quad I = \frac{h\rho R^4 \pi}{2} = \frac{MR^2}{2}$

M – total mass of liquid

work done $= \frac{MR^2\omega^2}{4}$

heat supplied: 0

temperature: T

at time ∞

work done: 0

heat supplied: $\frac{MR^2\omega^2}{4}$ (dissipation, collisions)

$$C(T_f - T) = \frac{MR^2\omega^2}{4}$$

$$\boxed{T_f = T + \frac{MR^2\omega^2}{4C}}$$