# PHYS-552: Advanced Statistical Mechanics 

April 30, 2012

Due date: 3rd of May, 2012

## 1 Number representation

- Given a set of creation and annihilation operators $c_{m 1}^{\dagger}, \ldots, c_{m L}^{\dagger}, c_{n 1}, \ldots, c_{n L}$, calculate the quantity $\langle 0| c_{n L} \ldots c_{n 1} c_{m 1}^{\dagger} \ldots c_{m L}^{\dagger}|0\rangle$ for both the bosonic and fermionic case.
- Given two identical fermions on a lattice of length $L$ in a state

$$
\begin{equation*}
|\Psi\rangle=c_{k 1}^{\dagger} c_{k 2}^{\dagger}|0\rangle, \tag{1}
\end{equation*}
$$

where $k_{1}$ and $k_{2}$ refer to plane waves. Show that if we switch to the position representation $|\Psi\rangle$ becomes

$$
\begin{equation*}
|\Psi\rangle=\frac{1}{L} \sum_{r 1, r 2}[\exp (i(k 1 r 1+k 2 r 2))-\exp (i(k 1 r 2+k 2 r 1))] c_{r 1}^{\dagger} c_{r 2}^{\dagger}|0\rangle \tag{2}
\end{equation*}
$$

## 2 2D Ising model

In class the high and low temperature representations of the two-dimensional Ising model were presented. Using these two representations derive the contribution to the free energy originating from the simplest closed polygon, i.e. a square surrounded by four bonds.

## 3 Rubber band with harmonic potential

A rubber band is suspended horizontally between points $y(0)=y(L)=0$. The partition function for a rubber band in a harmonic potential is given by

$$
\begin{equation*}
Z=\int_{y(0)=0}^{y(L)=0} D[y(x)] e^{-\beta \int \mathrm{d} x\left(A \dot{y}^{2}(x)+B y^{2}(x)\right)} . \tag{3}
\end{equation*}
$$

Calculate the average $\left\langle y(x) y\left(x^{\prime}\right)\right\rangle$.

