PHYS-552: Advanced Statistical Mechanics

April 30, 2012

Due date: 3rd of May, 2012

1 Number representation

- Given a set of creation and annihilation operators $c_{m1}^{\dagger}, ..., c_{mL}^{\dagger}, c_{n1}, ..., c_{nL}$, calculate the quantity $\langle 0|c_{nL}...c_{n1}c_{m1}^{\dagger}...c_{mL}^{\dagger}|0\rangle$ for both the bosonic and fermionic case.
- Given two identical fermions on a lattice of length L in a state

$$|\Psi\rangle = c_{k1}^{\dagger} c_{k2}^{\dagger} |0\rangle, \tag{1}$$

where k_1 and k_2 refer to plane waves. Show that if we switch to the position representation $|\Psi\rangle$ becomes

$$|\Psi\rangle = \frac{1}{L} \sum_{r1,r2} [\exp(i(k1r1 + k2r2)) - \exp(i(k1r2 + k2r1))]c_{r1}^{\dagger}c_{r2}^{\dagger}|0\rangle.$$
(2)

2 2D Ising model

In class the high and low temperature representations of the two-dimensional Ising model were presented. Using these two representations derive the contribution to the free energy originating from the simplest closed polygon, i.e. a square surrounded by four bonds.

3 Rubber band with harmonic potential

A rubber band is suspended horizontally between points y(0) = y(L) = 0. The partition function for a rubber band in a harmonic potential is given by

$$Z = \int_{y(0)=0}^{y(L)=0} D[y(x)] e^{-\beta \int dx (A\dot{y}^2(x) + By^2(x))}.$$
(3)

Calculate the average $\langle y(x)y(x')\rangle$.