# PHYS-552: Advanced Statistical Mechanics 

April 20, 2012

Due date: 26th of April, 2012

## 1 Ginzburg-Landau theory of tricritical points

A tricritial point is a point on a phase diagram which separates a first-order phase line from a second-order one. Consider a free energy

$$
\begin{equation*}
\Phi(T, Y, \eta)=\Phi_{0}(T, Y)+\alpha_{2} \eta^{2}+\alpha_{4}(T, Y) \eta^{4}+\alpha_{6}(T, Y) \eta^{6} \tag{1}
\end{equation*}
$$

Show that such a form for the free energy can give rise to a tricritical point.

## 2 Berthelot equation of state

An alternative to the van der Waals equation of state is the Berthelot equation, which reads

$$
\begin{equation*}
\left(P+\frac{a}{T v^{2}}\right)(v-b)=R T . \tag{2}
\end{equation*}
$$

Calculate the critical exponents of a system described by this equation of state.

## 3 Widom and Kadanoff Scaling

Considering a magnetic system, and assuming, as Widom has done, that the intensive free-energy can be written as

$$
\begin{equation*}
f(T, B)=t^{1 / y} \psi\left(B / t^{x / y}\right) \tag{3}
\end{equation*}
$$

where $t=\left|T-T_{c}\right| / T_{c}$. Based on the definitions of the critical exponents seen in class show that

$$
\begin{array}{r}
\alpha+2 \beta+\gamma=2 \\
\alpha+\beta(\delta+1)=2 \tag{5}
\end{array}
$$

From the Kadanoff scaling hypothesis for the two-point correlation function

$$
\begin{equation*}
G_{c}^{(2)}(r, t)=\frac{\psi\left(r t^{(2-\alpha) / d}\right)}{r^{d-2+\eta}} \tag{6}
\end{equation*}
$$

show that

$$
\begin{gather*}
(2-\eta) \nu=\gamma  \tag{7}\\
\nu d=2-\alpha \tag{8}
\end{gather*}
$$

## 4 Two leg Ising ladder model

The two leg Ising ladder can be defined as two one-dimensional Ising models placed parallel to each other. Nearest neighbor spins are coupled in the longitudinal direction, and each pair of spins is coupled in the transverse direction. Assuming that the coupling in each direction is $J$, calculate the transfer ma-


Figure 1: Ladder system.
trix associated with this system. See how far you can get in calculating the properties of this model, using the transfer matrix (partition function, energy, correlation function, etc.).

## 5 Renormalization for the 1D Ising model

In class you may have seen (at least it was presented) the renormalization transformation scheme for the 1D Ising model in which every second spin was eliminated. One could also block each pairs of spins into one spin and apply the following rule: if the two spins are parallel, assign the blocked spin a value of +1 , if they are anti-parallel assign -1 . Derive the renormalization transformation associated with such a blocking scheme. Find the associated fixed points.

