PHYS-552: Advanced Statistical Mechanics

March 9, 2012

Due date: 15th of March, 2012

1 Multivariant Gaussian distribution

Consider the probability density given by

$$P_{X_1,...,X_N}(x_1,...,x_N) = \sqrt{\frac{det(\mathbf{g})}{(2\pi)^N}} \exp\left\{-\frac{1}{2}\mathbf{x}^T \cdot \mathbf{g} \cdot \mathbf{x}\right\}, \tag{1}$$

where $\mathbf{g}$ is a symmetric $N \times N$ positive definite matrix, $\mathbf{x}$ is a column vector with $\mathbf{x}^T$ being the transpose of $\mathbf{x}$.

- Show that $P_{X_1,...,X_N}(x_1,...,x_N)$ is normalized to one.
- Compute the characteristic function $f_{X_1,...,X_N}(k_1,...,k_N)$.
- Show that the moments $\langle x_i \rangle = 0$ and that all higher moments can be expressed in terms of $\langle x_i^2 \rangle$ and $\langle x_i x_j \rangle$.

2 Thermal fluctuations in a rubber band

A rubber band is suspended between points 0 and L of equal height ($y(0) = y(L) = 0$). The height of the string at a particular point between 0 and L is given by the function $y(x)$. The energy ıs a functional of $y(x)$ and is given by

$$E[y(x)] = \frac{1}{2}\int_0^L \mathrm{d}x \left(\frac{\mathrm{d}y(x)}{\mathrm{d}x}\right)^2. \tag{2}$$

- Write the partition function of this system in the canonical ensemble as a functional integral over all possible functions $y(x)$.
- Represent the function $y(x)$ on a discrete set of points $x_0,...x_N$, where $x_0 = x_N = 0$. Write the canonical partition function in this discretized approximation.
- Calculate $\langle y(x_i)y(x_j)\rangle$, where $i < j$.

3 Pathria & Beale: Problem 4.2

4 Pathria & Beale: Problem 4.3

5 Pathria & Beale: Problem 4.4