PHYS-552: Advanced Statistical Mechanics

February 14, 2012

Due date: 20th of February, 2012

1 Efficiency of the Carnot cycle for an ideal gas

Compute the efficiency for the Carnot engine operating between a hot reservoir at temperature T_h and a cold reservoir at temperature T_c and using an ideal gas as its working substance. The equation of state of the ideal gas is

$$PV = Nk_BT, \tag{1}$$

and the internal energy is given by

$$E = \frac{3}{2}Nk_BT. \tag{2}$$

Express the answer in terms of the two temperatures.

2 Efficiency of a magnetic engine

A Carnot engine operating between a hot reservoir at temperature T_h and a cold reservoir at temperature T_c uses a paramagnetic substance as its working substance.

- Show that the internal energy, and therefore the heat capacity C_M is independent of the magnetization. In other words show that

$$\left(\frac{\partial E}{\partial M}\right)_{N,T} = 0. \tag{3}$$

In the following assume that C_M is constant.

- Sketch a typical cycle in the $M - H$ plane for the Carnot engine.
- Calculate the heat absorbed, the work done, and the efficiency of this engine. Express the latter in terms of temperatures.

The equation of state is $M = NDH/T$, where M denotes the magnetization, N denotes the size of the system, H denotes therefore magnetic field. D is a constant depending on material parameters.

3 Pathria and Beale: Problem 1.7

Study the statistical mechanics of an extreme relativistic gas characterized by the sintle-particle energy states

$$\epsilon(n_x, n_y, n_z) = \frac{hc}{2L}\left(n_x^2 + n_y^2 + n_z^2\right)^{\frac{1}{2}} \tag{4}$$

along the lines discussed in class or section 1.4 in the book. Show that the ration C_P/C_V in this case is 4/3.

4 Pathria and Beale: Problem 1.9

Making use of the fact that the entropy $S(N, V, E)$ of a thermodynamic quantity is extensive, show that

$$S = \left(\frac{\partial S}{\partial N}\right)_{V,E} N + \left(\frac{\partial S}{\partial V}\right)_{N,E} V + \left(\frac{\partial S}{E}\right)_{V,N} E. \tag{5}$$

5 Pathria and Beale: Problem 1.13

If the two gases considered in the mixing process of Section 1.5 were initially at different temperatures, say T_1 and T_2, what would the entopy of mixing be in that case? Would this contribution depend on whether the molecules are different or identical?