Quiz 4

hexagonal model with inversion symmetry

Haldane type Hamiltonian with inversion symmetry breaking term

$$\sum_{\vec{k}} F(\vec{k}) c_{\vec{k}}^{+} d_{\vec{k}} + d_{\vec{k}}^{+} c_{\vec{k}} F(\vec{k})$$

$$+ \sum_{\vec{k}} c_{\vec{k}}^{+} c_{\vec{k}} M(\vec{k}) + \sum_{\vec{k}} d_{\vec{k}}^{+} d_{\vec{k}} M(\vec{k})$$

$$\underbrace{+ \mu \sum_{\vec{k}} c_{\vec{k}}^{+} c_{\vec{k}} - \mu \sum_{\vec{k}} d_{\vec{k}}^{+} d_{\vec{k}}}_{\text{breaks inversion symmetry}}$$

$F(\vec{k}) = \sum_{i=1}^{3} e^{i\vec{k}\cdot\vec{r}_i^{(1)}}$ → nearest neighbors $(\vec{r}_i^{(1)})$

$M(\vec{k}) = \sum_{i=1}^{3} e^{i\vec{k}\cdot\vec{r}_i^{(2)}}$ → second nearest neighbors $(\vec{r}_i^{(2)})$

Dirac points: $\vec{K}_1, \vec{K}_2 \Rightarrow \vec{K}_2 = -\vec{K}$

~~$F(\vec{k}) \rightarrow$~~ sending $\vec{k} \rightarrow -\vec{k}$; $F(\vec{k}) \rightarrow F^*(\vec{k})$

$M(\vec{k}) \rightarrow M^*(\vec{k})$

$\mu \rightarrow \mu$

write Hamiltonian in "two-state" form ②

$$\vec{h}(\vec{k})\cdot\vec{\sigma} = h\begin{pmatrix} \cos\theta & \sin\theta \cancel{\sin\theta} e^{i\varphi} \\ \sin\theta e^{-i\varphi} & -\cos\theta \end{pmatrix} = \begin{pmatrix} h_z & h_x + ih_y \\ h_x - ih_y & -h_z \end{pmatrix}$$

diagonalise:

$$\begin{pmatrix} \cos\theta - \lambda & \sin\theta e^{i\varphi} \\ \sin\theta e^{-i\varphi} & -\cos\theta - \lambda \end{pmatrix}$$

$$\Rightarrow -(\cos\theta - \lambda)(\cos\theta + \lambda) - \sin^2\theta = 0$$

$$\lambda^2 = \cos^2\theta + \sin^2\theta \Rightarrow \lambda = \pm 1$$

~~cos θ~~ eigenvector of state with $\lambda = -1$

$$(\cos\theta + 1)x + \sin\theta e^{i\varphi} y = 0$$

$$x = 1$$

$$y = -\frac{(\cos\theta + 1)}{\sin\theta\, e^{i\varphi}}$$

$$y = -\cancel{\cos\tfrac{\theta}{2}\cos\tfrac{\theta}{2} - \sin\tfrac{\theta}{2}\sin\tfrac{\theta}{2} + \cos\tfrac{\theta}{2}\cos\tfrac{\theta}{2}}$$

$$y = -\frac{\cos\frac{\theta}{2}\cos\frac{\theta}{2} - \cancel{\sin\frac{\theta}{2}\sin\frac{\theta}{2}} + \cos\frac{\theta}{2}\cos\frac{\theta}{2} + \cancel{\sin\frac{\theta}{2}\sin\frac{\theta}{2}}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\, e^{i\varphi}}$$

$$= -\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}\, e^{i\varphi}}$$

$$\rightarrow u_- = \begin{pmatrix} \sin\frac{\theta}{2} e^{i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

Berry connection: $\cancel{A} \cdot i\langle u_- | \partial_\theta | u_- \rangle = \frac{i}{2}\overbrace{\left[\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]}^{=0}$

$$i\langle u_- | \partial_\varphi | u_- \rangle = -\sin^2\frac{\theta}{2}$$

Berry curvature: at $\vec{K}$

$$\boxed{\begin{aligned} i\partial_\theta \langle u_- | \partial_\varphi | u_- \rangle &= -\sin\tfrac{\theta}{2}\cos\tfrac{\theta}{2} \\ &= -\frac{\sin\theta}{2} \end{aligned}}$$

send $\vec{K} \to -\vec{K}$ for model with inversion symmetry breaking term means

$$\varphi \to -\varphi$$

since
$$h_z \to h_z$$
$$h_x \to h_x$$
$$h_y \to -h_y$$

in this case

$$u_- = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\varphi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

$$\Rightarrow \boxed{\text{curvature at } -\vec{K} \quad +\frac{\sin\theta}{2}}$$

changes sign! Berry phases have to cancel

for Haldane model with time-reversal invariance breaking term

$$h_x \to h_x$$
$$h_y \to -h_y$$
$$h_z \to -h_z$$

$$\varphi \to -\varphi$$
$$\theta \to \pi - \theta$$

$$\boxed{\text{at } \vec{K}: \text{ curvature is } -\frac{\sin\theta}{2}}$$

Hamiltonian becomes

$$h \begin{pmatrix} -\cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & \cos\theta \end{pmatrix}$$

~~diago~~

$\cancel{(\cos\theta - \lambda)}$

diagonalize: $(-\cos\theta - \lambda)(\cos\theta - \lambda) - \sin^2\theta = 0$

$$-(\cos^2\theta - \lambda^2) - \sin^2\theta = 0$$

$$\lambda = \pm 1 \times h$$

eigenvector w/ $\lambda = -h$

$\Rightarrow \quad (-\cos\theta + 1)x + \sin\theta e^{-i\varphi} y = 0$

$$x = 1$$

$$\Rightarrow y = \frac{(\cos\theta - 1)}{\sin\theta e^{-i\varphi}}$$

$\cos\theta - 1 = \cos\frac{\theta}{2}\cancel{\cos\frac{\theta}{2}}\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\sin\frac{\theta}{2} \cancel{- \cos\frac{\theta}{2}}\cos\frac{\theta}{2}$

$$- \sin\frac{\theta}{2}\sin\frac{\theta}{2}$$

$$y = \frac{-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} e^{-i\varphi}}$$

$\Rightarrow$ vector: $\begin{pmatrix} \cos\frac{\theta}{2} e^{-i\varphi} \\ -\sin\frac{\theta}{2} \end{pmatrix}$

Berry connection $i\langle u_- | \partial_\theta | u_- \rangle = 0$

$$i\langle u_- | \partial_\varphi | u_- \rangle = \cos^2\frac{\theta}{2}$$

Berry curvature: $-2\cos\frac{\theta}{2}\sin\frac{\theta}{2} = -\frac{\sin\theta}{2}$

at $-\hat{k}$