PHYS 371 HW II

Analytical Part Solutions

2.1. We need two Taylor expansions to derive the three-point forward difference formula

$$f(t+\tau) = f(t) + \tau f'(t) + \frac{\tau^2}{2} f''(t) + o(\tau^3) \qquad (a)$$

$$f(t+2\tau) = f(t) + 2\tau f'(t) + \frac{4\tau^2}{2} f''(t) + o(\tau^3) \qquad (b)$$

$$4a - b \Rightarrow 4f(t+\tau) - f(t+2\tau) = 3f(t) + 2\tau f'(t) + O(\tau^3)$$

reorganizing we have:

$$\boxed{f'(t) = \frac{4f(t+\tau) - f(t+2\tau) - 3f(t)}{2\tau} + o(\tau^2)}$$

2.22

$$\left.\begin{aligned} \vec{r}_{n+1} &= \vec{r}_n + \tau \vec{v}_n + \frac{1}{2}\tau^2 \vec{a}_n \\ \vec{v}_{n+1} &= \vec{v}_n + \frac{1}{2}\tau(\vec{a}_n + \vec{a}_{n+1}) \end{aligned}\right\} \text{velocity Verlet}$$

$$\left.\begin{aligned} \vec{v}_n &= \frac{\vec{r}_{n+1} - \vec{r}_{n-1}}{2\tau} + o(\tau^2) \\ \vec{r}_{n+1} &= 2\vec{r}_n - \vec{r}_{n-1} + \tau^2 \vec{a}_n + o(\tau^4) \end{aligned}\right\} \text{standart Verlet}$$

$\vec{r}_{n-1} = \vec{r}_{n+1} - 2\tau \vec{v}_n \rightarrow$ use this in equation for $\vec{r}_{n+1}$

$$\vec{r}_{n+1} = 2\vec{r}_n - \vec{r}_{n+1} + 2\tau \vec{v}_n + \tau^2 \vec{a}_n$$

$$\Rightarrow \boxed{\vec{r}_{n+1} = \vec{r}_n + \tau \vec{v}_n + \frac{1}{2}\tau^2 \vec{a}_n}$$

so the $\vec{r}_n$'s are the same for both methods