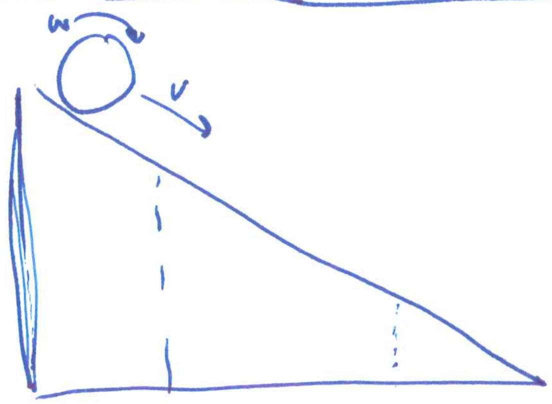
Problem Set 8: Solutions

1.)



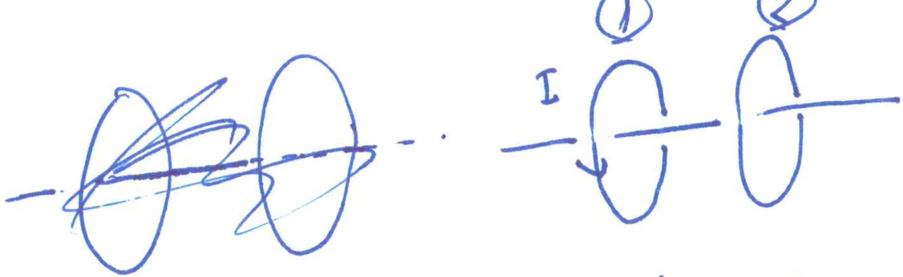
motion consists of two components:

- rotation
- translation

- translation: component parallel to v ⇒ no change
  component perpendicular to v ⇒ force will act sideways
- rotation: component parallel to ω ⇒ no change
  component perpendicular to ω ⇒ sets up potential difference between center and edge

2.) Yes. The magnetic field, and hence the flux, will vary as a function of time.

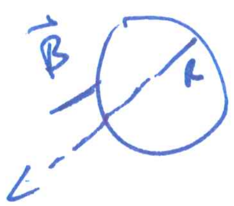
3.)



1.) magnetic field points in z ← direction
induced current will oppose: ⇒ it will be counterclockwise (opposite direction to loop 1)

2.) force will be repulsive

4.)



ρ – resistivity

$B = C_1 t + C_2 t^2$

current as a fn. of time

$$\varepsilon(t) = -\frac{d\Phi_B}{dt} = -\frac{d(C_1 t + C_2 t^2)\pi R^2}{dt}$$

$$= -(C_1 + 2C_2 t)\pi R^2$$

to obtain current need resistance

$\tilde{R} = \frac{\rho 2\pi R}{A}$ A = area of wire

y

$$I(t)\, \rho \frac{2\cancel{\pi} \cancel{R}}{A} = -(C_1 + 2C_2 t)\cancel{\pi} R^{\cancel{2}}$$

$$\boxed{I(t) = -\frac{A(C_1 + 2C_2 t)}{\rho}}$$

5.)

resistance: R

$$\cancel{\frac{dB}{dt} = \frac{B}{\Delta t}}$$

$$\cancel{\text{flux } \Phi_B = B\pi R^2 A}$$

$$\cancel{\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{B\pi R^2}{\Delta t} = IR}$$

$$\cancel{I = -\frac{B\pi R^2}{R\Delta t}}$$

flux $\Phi_B = B \cdot A$

suppose drop is linear in time

$$B(t) = B\left(1 - \frac{t}{\Delta t}\right)$$

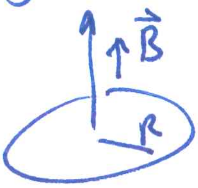
$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{BA}{\Delta t} = IR$$

$$I = \frac{BA}{\Delta t R}$$

Power dissipated $P = \frac{B^2 A^2}{\Delta t^2 R}$ $(= I^2 R)$

energy dissipated $E = \frac{B^2 A^2}{\Delta t R}$

6.)



$\vec{B}(t)$

net charge: $Q(t) = \int_0^t I(t')\,dt'$

given point: $\frac{Q(t)}{A}$ (charge over area)

$$\Phi_B = B(t)\pi R^2$$

$$\mathcal{E} = -\frac{dB(t)}{dt}\pi R^2$$

resistance of loop: $\tilde{R} = \frac{\rho L}{A} = \frac{\rho 2\pi R}{A}$

$$\mathcal{E} = I(t)\tilde{R} = I(t)\frac{\rho 2\pi R}{A} = -\frac{dB(t)}{dt}\pi R^2$$

$$I(t) = -\frac{dB(t)}{dt}\frac{AR}{2\rho} = -\frac{dB(t)}{dt}$$

$$Q(t) = \frac{AR}{2\rho}(B(t) - B(0))$$

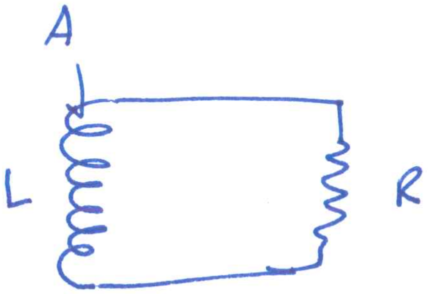
(4)

across single point

$$q(t) = \frac{Q(t)}{A} = \frac{R}{2s}\left(B(t) - B(0)\right)$$

2. No – it can oscillate around zero (pass through point, and go backwards)

7.)



$$\mathcal{E} = -\frac{d N \Phi_s}{dt}$$

$$= -NA\frac{dB(t)}{dt} = I(t)\,R$$

$$-NA\frac{dB(t)}{dt} = \frac{dQ(t)}{dt}R$$

integrate both sides

$$-NA\left[B(t) - B(0)\right] = \left[Q(t) - Q(0)\right]R$$

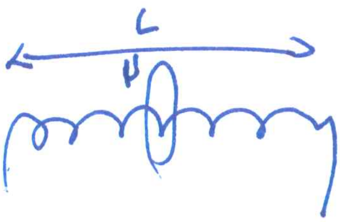
B goes from $B \to -B$

$$B(0) = B$$

$$B(t) = -B$$

$$\boxed{\frac{2NAB}{R} = \Delta Q}$$

8.) magnetic field inside solenoid



N turns per unit length

$$B \cdot L = \mu_0 N L I$$

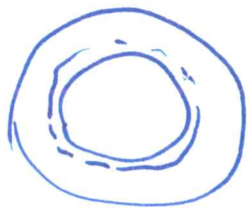
$$B = \mu_0 N I$$

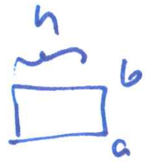
$$B(t) = \mu_0 N \cancel{L}\left(I + \frac{(I' - I)}{T}t\right)$$

$$B(t) = \mu_0 N \left(I + \frac{(I' - I)t}{T} \right)$$

$$\boxed{\mathcal{E} = -\frac{dB(t)}{dt} \pi R^2 = \mu_0 N \frac{(I' - I)}{T} \pi R^2}$$

9.) flux through toroid





$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$\vec{B}$ - obtain from Ampère's law

$$B(r)\, 2\pi r = \mu_0 N I$$

$$B(r) = \frac{\mu_0 N I}{2\pi r}$$

$$\Phi_B = \int_a^b \frac{\mu_0 N I}{2\pi r} \cdot h\, dr$$

$$\boxed{\Phi_B = \frac{\mu_0 N I h}{2\pi} \ln(b/a)}$$

(10)

flux

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

area of loop: $\frac{x^2(t)}{\cancel{4}}$

$\Phi_B = B\, x^2(t)$

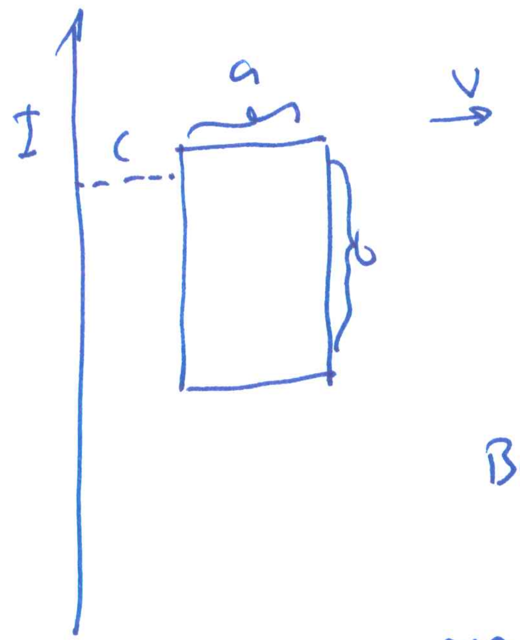
$\varepsilon = -B\, 2x(t)\, v(t)$

(11) $\Phi_B = BA\cos\omega t = B\pi a^2 \cos\omega t$

$\varepsilon = \frac{-d\Phi_B}{dt} = B\pi a^2 \omega \sin\omega t$

current: $\frac{B\pi a^2 \omega \sin\omega t}{R}$

(12)



$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$= b \int_c^{c+a} B(r)\, dr$

$B(r)\, 2\pi r = I$

$B(r) = \frac{I}{2\pi r}$

$\Phi_B = b \int_c^{c+a} \frac{I}{2\pi r}\, dr = \frac{bI}{2\pi} \ln\left(\frac{c+a}{c}\right)$

current $\mathcal{E} = -\frac{d\Phi_B}{dt}$

only c is changing in time $(c(t)=c)$

$=$

$$\mathcal{E} = -\frac{\mu_0 I}{2\pi}\left[\frac{c(t)}{c(t)+a}\right]\cancel{\frac{dc}{dt}}\left[\frac{1}{c(t)} - \frac{(c(t)+a)}{c^2(t)}\right]\frac{dc(t)}{dt}$$

$\mathcal{E} = I_\varphi R \Rightarrow I_\varphi(t)$

$$\boxed{I_\varphi(t) = -\frac{\mu_0 I}{2\pi}\frac{a\,\dot{c}}{(a+c)\,c\,R}}$$