Solutions: Problem Set 6

1.) 1.) $V_{ab} = 70V$

$A = 2.6 \times 10^{-6}\ m^2$

$\rho = 5.0 \times 10^{-7}\ \Omega m$

1. $P = 4500\ W = I^2 R$

$d = ?$

$V_{ab} = I R \qquad \frac{d\rho}{A} = R$

$$P = \frac{V_{ab}^2}{R} = \frac{V_{ab}^2 A}{d\rho} \Rightarrow \boxed{d = \frac{V_{ab}^2 A}{\rho P}}$$

$$d = \frac{70^2 \times 2.6 \times 10^{-6}}{5.0 \times 10^{-7} \times 4500} = \frac{49\cancel{00} \times 2.6 \times \cancel{10}\,2}{\cancel{5.0} \times 45\cancel{00}}$$

$$= \frac{49 \times 5.2}{45} = \frac{254.8}{45} = 254.8 \div 45 =$$

$$= 2548 \div 450 = \underline{\underline{5.662}}\ m$$

2980
2800
1000

2.) $$d = \frac{(100)^2 A}{\rho P} = \frac{(100)^2 \times 2.6 \times \cancel{10}^{-6}}{5.0 \times \cancel{10}^{-7} \times 4500} = \frac{\overset{20}{\cancel{100\cancel{00}}} \times 2.6}{\cancel{5} \times 45\cancel{00}}$$

$$= \frac{520}{45} = 520 \div 45 = \underline{\underline{11.\overline{5}\,m}}$$

70
250
25

2.) $P = 100\,W$

$V = 220\,V$

1.) 1 month : 30 days

$30 \cdot 24 = 720\ hr.$

60
120
720

1 month: energy consumed => 72000 Whr

72 kW·hr

=> $72 \cdot 15$ kurs

72
360
1080

$\boxed{10.80\,TL}$

2.) $P = I^2 R = IV \Rightarrow \dfrac{100\,W}{220\,V} = I$

$V = IR$

$V = \dfrac{100}{220} R = 220\,V \Rightarrow \dfrac{(220)^2}{100} = R = 484\,\Omega$

$220 \cdot 220$
440
4400
48400
~~00~~

3.) $\dfrac{100}{220} = 100 \div 220 = 10\cancel{0} \div 22 = 0.45\ A$

100
120

4.) Yes because usually the resistance depends on temperature

3.) $E = 15\,MeV$ per particle

$I = 15\,\mu A$

1.) rate of particles striking block ⇒ $\underline{F}$

$I = 15\,\mu A = 15\,\frac{\mu C}{s}$

$1e = 1.602 \times 10^{-19}\,C \sim 1.6 \times 10^{-9}$

$\cancel{F / 2 = I}$

deuteron has charge $+1e$

$\Rightarrow F = \frac{I}{1.6 \times 10^{-19}} \sim 9.4 \times 10^{13}/s$

2.) rate of energy production

$F = 9.4 \times 10^{13}$ particles/second

each particle has 15MeV energy

$P = 15 \cancel{*} 9.4 \times 10^{13}\,\frac{MeV}{s} = 225\,W$

4.) 9V electron has charge $-1e$

1.) $1e = 1.6 \times 10^{-19}$

work done: $\frac{1.6 \times 9}{14.4} \Rightarrow 14.4 \times 10^{-19}\,J$

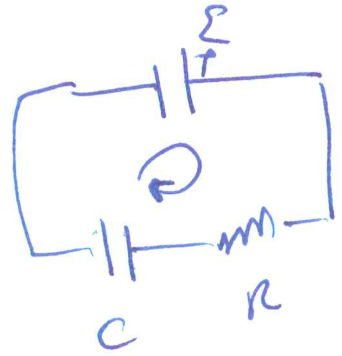
1.44×10^{-18} J/particle

2.) 3.4×10^{18} electrons/second

$P = 1.44 \times 10^{-18} \cdot 3.4 \times 10^{+18}\,\frac{J}{s}$

$= 4.9\,W$

5.)



$$\varepsilon - IR - \frac{Q}{C} = 0 \qquad I = \frac{dQ}{dt}$$

$$\varepsilon - \frac{dQ}{dt}R - \frac{Q}{C} = 0$$

$$\frac{\varepsilon}{R} - \frac{Q}{RC} = \frac{dQ}{dt}$$

$$\frac{\varepsilon C - Q(t)}{RC} = \frac{dQ}{dt}$$

$$\frac{dt}{RC} = \frac{dQ}{\varepsilon C - Q(t)}$$

$$-\frac{dt}{RC} = \frac{dQ}{Q(t) - \varepsilon C}$$

integrate ⇒ $$-\frac{t}{RC}\Big|_{t=t_i}^{t=t_f} = \ln\left[Q(t) - \varepsilon C\right]\Big|_{t=t_i}^{t=t_f}$$

$$e^{-\frac{t_f - t_i}{RC}} = \frac{Q(t_f) - \varepsilon C}{Q(t_i) - \varepsilon C}$$

if at $t_i = 0$ $Q(t_i) = 0$

$$\Rightarrow e^{-\frac{t}{RC}} = \frac{Q(t) - \varepsilon C}{-\varepsilon C} = \frac{\varepsilon C - Q(t)}{\varepsilon C}$$

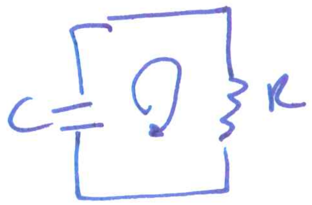
time constant → $\tau = RC$

1.) $\tau = (1.4)^2\ s = 1.96$ sec

2.) maximum charge $t \to \infty \Rightarrow Q_e = \varepsilon C \to 1.7 \times 10^{-5}$ C

5.3.) $G(0) = \varepsilon C$

battery removed



$$IR + \frac{Q}{C} = 0$$

$$\frac{dQ}{dt}R = -\frac{Q}{C} \Rightarrow \frac{dQ}{Q} = -\frac{dt}{RC}$$

$$\cancel{-dt} \qquad \cancel{\ln Q = e^{-t/RC}}$$

$$\cancel{\ln}\; Q(t)\Big|_0^t = e^{-t/RC}$$

$$\cancel{\ln}\; \boxed{Q(t)/Q_0 = e^{-t/RC}}$$

one-half of original charge

$$\frac{Q(t)}{Q_0} = \frac{1}{2} \Rightarrow e^{-t/RC} = 1/2$$

$$\ln e^{-t/RC} = -\ln 2$$

$$+\frac{t}{RC} = +\ln 2$$

$$t = RC \ln 2 = 1.96 \cdot \ln 2 \approx 1.36 \text{ sec.}$$

6.)

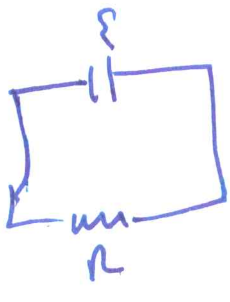
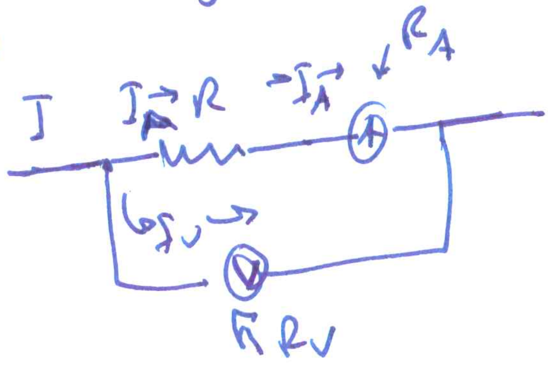


Fig. 26.16a



6.) force

(1) ~~$I\,d\vec{l}$~~ $d\vec{F} = I\, d\vec{l} \times \vec{B}(\vec{l}) \qquad d\vec{l} = dy$

$$= I \cancel{dy} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & dy & 0 \\ 0 & \frac{B_0 z}{L} & \frac{B_0 y}{L} \end{vmatrix}$$

$$\vec{F} = I \frac{B_0}{L} \int_0^L y\, dy\, \hat{i} = \frac{I B_0}{L} \frac{L^2}{2} \hat{i} = \frac{I B_0 L}{2} \hat{i}$$

direction: positive x

(2) $d\vec{F} = I\, d\vec{l} \times \vec{B}(\vec{l})$

$$= I \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & 0 & 0 \\ 0 & \frac{B_0 z}{L} & \frac{B_0 y}{L} \end{vmatrix} \qquad z = 0 \quad y = L$$

$$d\vec{F} = I \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ dx & 0 & 0 \\ 0 & 0 & B_0 \end{vmatrix} = -\hat{j}\, dx\, B$$

$\vec{F} = -I B_0 L \hat{j}$ direction: negative y

(3) $d\vec{F} = I\, d\vec{l} \times \vec{B}$

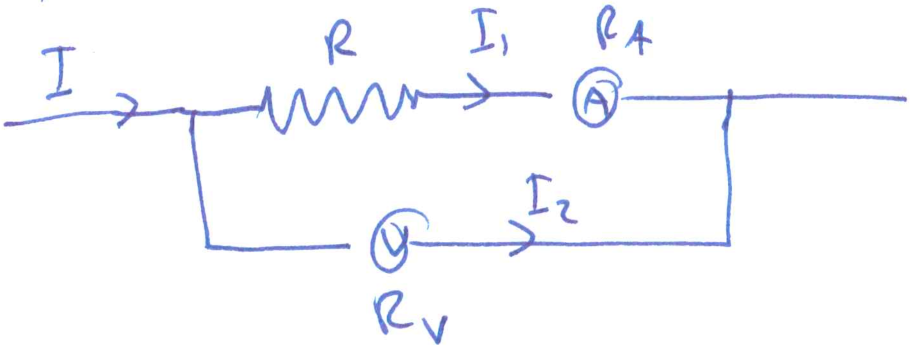
$$= I \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & dy & 0 \\ 0 & 0 & \frac{B_0 y}{L} \end{vmatrix} = -\frac{I B_0}{L} \int_L^0 dy\, y = -\frac{I B_0 L}{2} \hat{i}$$

(4) $z = 0,\ y = 0 \Rightarrow \vec{B} = (0, 0, 0)$

$\Rightarrow \vec{F} = 0$

net force $-I B_0 L \hat{j}$

6.) 1.) 26.16a



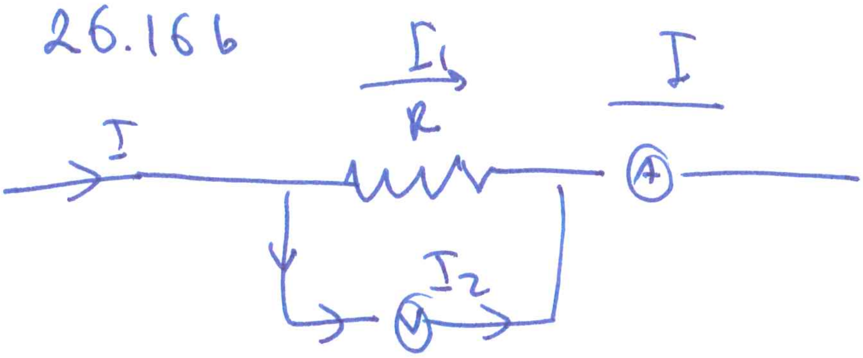
meter A reads: $I_1 \rightarrow I_{reading}$

meter V reads: $I_1 R + I_1 R_A \Rightarrow V_{reading}$

apparent resistance: $R' = \frac{V_{reading}}{I_{reading}}$

$$R' = R + R_A$$

2.) 26.16b



meter A reads: $I = I_1 + I_2 = I_{reading}$

meter V reads $I_1 R = V_{reading}$

apply Kirchoff's loop rule

$$I_1 R - I_2 R_V = 0$$

$$\Rightarrow V_{reading} = I_1 R = I_2 R_V$$

apparent resistance: $R' = \frac{V_{reading}}{I_{reading}}$

$$R' = \frac{I_1 R}{I_1 + I_2} = \frac{R}{1 + I_2/I_1} = \frac{R}{1 + R/R_V} = \frac{R R_V}{R + R_V}$$

7.) $I = 20A$ $\qquad \lambda = 46.6\,g/m = .0466\,kg/m$

$\vec{F} = I\,\vec{L} \times \vec{B}$

wire $\xrightarrow{I = 20A}$

if direction of $\vec{B}$-field is $\otimes$
force will act upwards

$F = ILB = mg$ $\qquad g$ = gravitational constant

mass $m = \lambda L$

$ILB = \lambda L g$

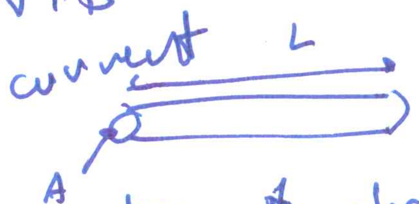
$IB = \lambda g$

$B = \frac{\lambda g}{I} = \frac{.0466 * 9.8}{20} =$

$B = .022834\,T$

8.) $\vec{F} = q\,\vec{v} \times \vec{B}$

wire,



n - number of charge carriers / volume

total force $n \times$ volume

$\vec{F} = neV\,\vec{v} \times \vec{B} =$

$= neAL\,\vec{v} \times \vec{B} =$

since L and $\vec{v}$ are parallel

$\Rightarrow neAv\,\vec{L} \times \vec{B} = I\,\vec{L} \times \vec{B}$

9.) energy of ions after passing through potential difference

$10kV \times 2e = 20 keV$

$= 20\,000\ eV$

$E = 20000 \times (1.6 \times 10^{-19})\ J = 3.2 \times 10^{-15}\ J$

mass of berillium atom 9 a.m.u

$9 \times 1.67 \times 10^{-27}\ kg = 15.03 \times 10^{-27}\ kg$

$= 1.5 \times 10^{-26}\ kg$

$\frac{1.67 \times 9}{15.03}$

velocity of atoms

$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{6.4 \times 10^{-15}}{1.5 \times 10^{-26}}} = \sqrt{\frac{6.4 \times 10^{4}}{1.5}}$

$v = 6.5 \times 10^{5}\ m/s$

$B = 1.2T$

$F = q\vec{v} \times \vec{B}$

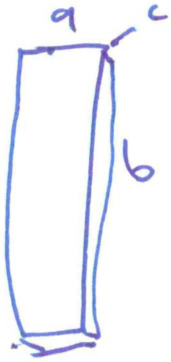
to balance we need $q\vec{E}$

$\vec{E} = \vec{v} \times \vec{B} \Rightarrow E = vB$

$E = 1.2 \times 6.5 \times 10^{5}\ \frac{J}{C}$

$= 7.8 \times 10^{5}\ J/C$

10.)



$a = 0.85\,cm$

$b = 6.5\,cm$

$c = 0.46\,mm$

potential difference across $a = 0.85\,cm = .0085\,m = 8.5\times10^{-3}\,m$

$$V = 3.9\,\mu V = Ea = E(.0085\,m)$$

$$V = 3.9\times10^{-6}\,V = E(.0085\,m)$$

$$E = \frac{3.9\times10^{-6}}{8.5\times10^{-3}}\,\frac{V}{m} = .46\times10^{-3} = 4.6\times10^{-4}\,\frac{V}{m}$$

$$\vec{E} = \vec{v}\times\vec{B} \rightarrow E = vB$$

$$v = \frac{4.6\times10^{-4}}{1.2\,mT}\,\frac{m}{s} = \frac{4.6\times10^{-4}}{1.2\times10^{-3}}$$

$$= \frac{4.6}{1.2}\times10^{-1}\,\frac{m}{s} = .38\,\frac{m}{s}$$

11.) 1.) m_p - mass of proton

$m_D = 2m_p$ - mass of deuteron

$m_\alpha = 4m_p$ - mass of α-particle

Charges

$q_p = +e$

$q_D = q_p$

$q_\alpha = 2q_p$

- kinetic energies: same for proton, deuteron

~~$T = qV$~~ twice as much for α-particle

2.) $$E_p = \frac{m_p v_p^2}{2}$$

$$E_D = E_p = \frac{m_D v_p^2}{2} = m_p v_D^2$$

$$\rightarrow v_D^2 = \frac{v_p^2}{2} \Rightarrow v_D = \frac{v_p}{\sqrt{2}}$$

$$E_\alpha = 2E_p = \frac{m_\alpha v_\alpha^2}{2} = \frac{4m_p v_\alpha^2}{2} \Rightarrow v_\alpha = \cancel{\ } \sqrt{\frac{E_p}{m_p}} = \frac{v_p}{\sqrt{2}}$$

11.2.) radius given by

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{\cancel{qm}\, v}{qB}$$

$$r_{H} = r_p = \frac{m_p v_p}{q_p B}$$

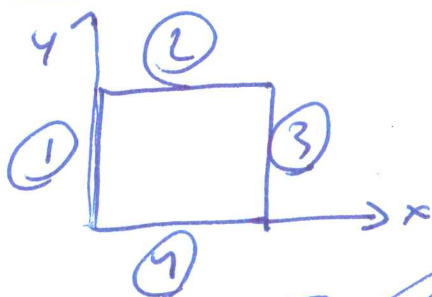
$$r_D = \frac{m_D v_D}{q_D B} = \frac{2 m_p v_p}{\sqrt{2}\, q_p B} = \sqrt{2}\, r_p$$

$$r_\alpha = \frac{m_\alpha v_\alpha}{q_\alpha B} = \frac{(4 m_p)(v_p \sqrt{2})}{(2 q_p) B} = \sqrt{\ }\, r_p$$

$r_D = \sqrt{2}\, r_p = 14$ cm

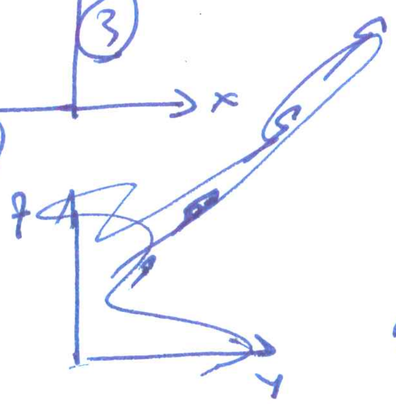
$r_\alpha = \sqrt{\ }\, r_p = 1\cancel{4}$ cm

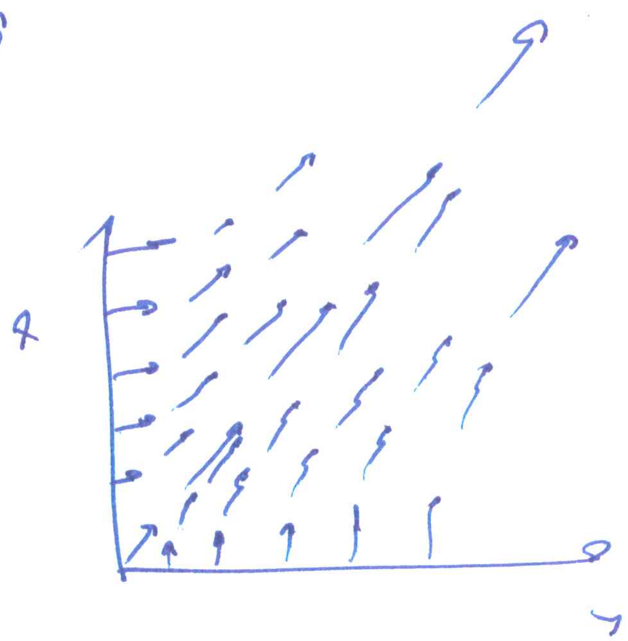
12.) $\underline{27.85}$



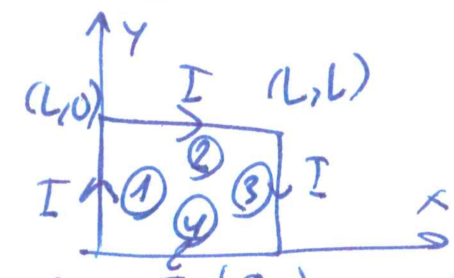
$$\vec{B} = \left(\frac{B_0 z}{L}\right)\hat{j} + \left(\frac{B_0 y}{L}\right)\hat{k}$$

(a)

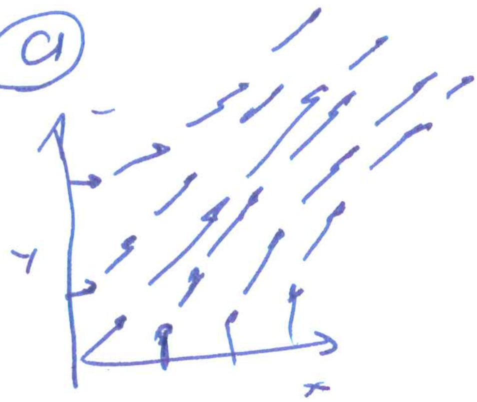




13.) 27.86



(a)



$$\vec{B} = \left(\frac{B_0 y}{L}\right)\hat{\imath} + \left(\frac{B_0 x}{L}\right)\hat{\jmath}$$

(b)

force (1):

$$d\vec{F} = I\, d\vec{L} \times \vec{B}$$

$$= I \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & dy & 0 \\ \frac{B_0 y}{L} & 0 & 0 \end{vmatrix} \qquad x = 0$$

$$dF_x = 0 \qquad dF_y = 0 \qquad dF_z = -\frac{I B_0}{L} y\, dy$$

$$F_x = 0 = F_y \quad F_z = -\frac{I B_0}{L} \int_0^L y\, dy = -\frac{I B_0 L}{2}$$

(2) $d\vec{F} = I\, d\vec{L} \times \vec{B}$

$$= I \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ dx & 0 & 0 \\ B_0 & \frac{B_0 x}{L} & 0 \end{vmatrix} =$$

$$dF_x = 0 = dF_y \qquad dF_z = \frac{I B_0}{L} \int_0^L x\, dx \Rightarrow \frac{I B_0 L}{2} = F_z$$

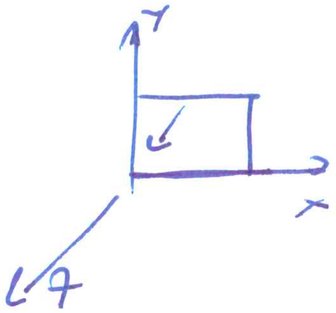
(3) $d\vec{F} = I\, d\vec{L} \times \vec{B} = I \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & dy & 0 \\ \frac{B_0 y}{L} & B_0 & 0 \end{vmatrix}$

$$F_z = \int_L^0 \left(-\frac{I B_0}{L}\right) y\, dy = \frac{I B_0 L}{2}$$

(4) $d\vec{F} = I\, d\vec{L} \times \vec{B}$ $\quad I \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ dx & 0 & 0 \\ 0 & \frac{B_0 x}{L} & 0 \end{vmatrix}$

$$F_z = \frac{I B_0}{L} \int_L^0 x\, dx = -\frac{I B_0 L}{2}$$

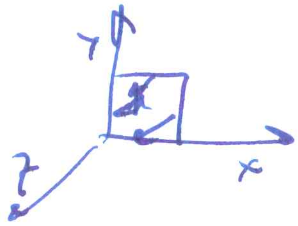
c) torque if free to rotate around x-axis



$$|\tau| = L \times F = \frac{I B_0 L^2}{2}$$

direction +x

d) torque if free to rotate around y-axis



$$|\tau| = \frac{I B_0 L^2}{2}$$

direction −y

e.) $\vec{\tau} = \vec{\mu} \times \vec{B}$ not appropriate

$\Rightarrow$ axis not in center