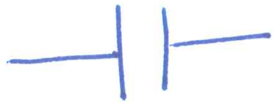
# Problem Set 5: Solutions



1.) if the capacitor is <u>designed</u> to withstand a certain potential but does not => the liquid material w/ dielectric constant $\kappa$ does not fill the volume of capacitor => Solution: fill it up! (liquids can also be compressed)

2.) 1.) capacitance



area: $A$
distance: $d$

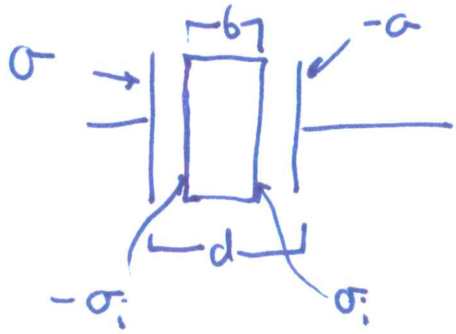
$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

$$V = Ed = \frac{Qd}{\varepsilon_0 A} = \frac{Q}{C} \Rightarrow C = \frac{\varepsilon_0 A}{d}$$

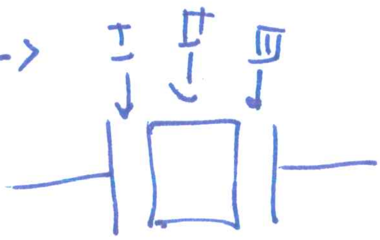
2.) free charge:

$$V = \frac{Q}{C} \longrightarrow Q = CV = \frac{\varepsilon_0 A V}{d}$$

3.) electric field with dielectric




regions




1.) $E_{I} = \frac{\sigma}{\varepsilon_0}$

2.) $E_{II} = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_i}{\varepsilon_0} = \cancel{\frac{\sigma}{\varepsilon_0}(1-\kappa)}$

$$\sigma_i = \sigma\left(1 - \frac{1}{\kappa}\right)$$

$$= \frac{\sigma}{\varepsilon_0} - \frac{\sigma}{\varepsilon_0}\left(1 - \frac{1}{\kappa}\right) = \frac{\sigma}{\kappa\varepsilon_0} - \frac{\sigma}{\varepsilon_0}$$

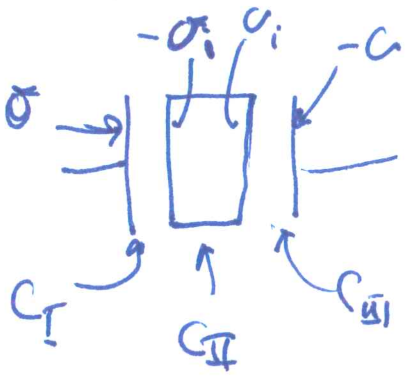
3.) $E_{III} = \frac{\sigma}{\varepsilon_0}$

2.4. potential difference

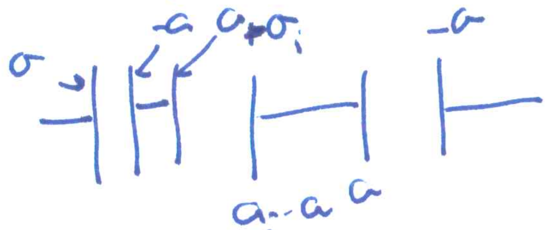
$$V = E_{I}\frac{(d-b)}{2} + E_{III}\frac{(d-b)}{2} + E_{II}\, b$$

$$= \frac{\sigma}{\varepsilon_0}(d-b) + \frac{\sigma}{\kappa\varepsilon_0} b = \frac{\sigma}{\varepsilon_0} d + \frac{\sigma b}{\varepsilon_0}\left(1 - \frac{1}{\kappa}\right)$$

2.5. Capacitance ~~of~~ with slab in place




decompose into:




three capacitors in series

$$C_{I} = \cancel{\frac{\varepsilon_0 (d-b)}{2A}} = \frac{\varepsilon_0 2A}{d-b} \qquad C_{II} = \cancel{\frac{\varepsilon b}{A}} = \frac{\varepsilon_0 A}{b} \qquad C_{III} = \cancel{\frac{\varepsilon_0 (d-b)}{2A}} = \frac{\varepsilon_0 2A}{d-b}$$

$$C_{II} = \frac{\kappa \varepsilon_0 A}{b}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_{I}} + \frac{1}{C_{II}} + \frac{1}{C_{III}} = \frac{1}{\varepsilon_0 A}\left[(d-b) + \frac{b}{\kappa}\right]$$

3.1 conductor between plates

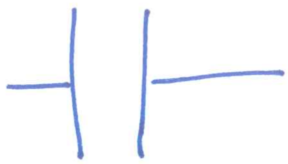
1.) $C = \frac{\varepsilon_0 A}{d}$

2.) $Q = CV$

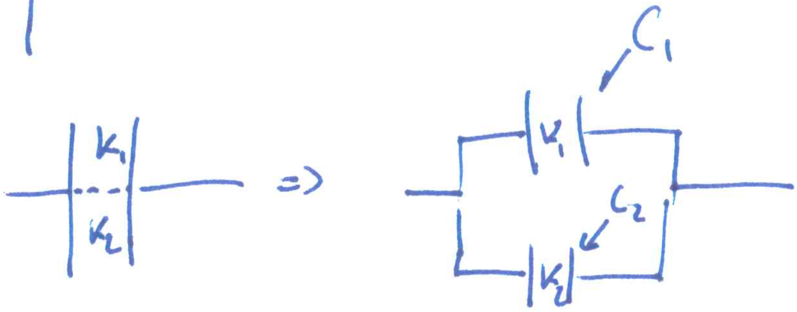
3.) $E_{I} = \frac{\sigma}{\varepsilon_0}$ $\quad E_{II} = 0$ $\quad E_{III} = \frac{\sigma}{\varepsilon_0}$

4.) $V = E\frac{(d-b)}{2} + E\frac{(d-b)}{2}$ $\quad$ 5.) $\frac{1}{C_{eq}} = \frac{1}{C_{I}} + \frac{1}{C_{II}} = \frac{d-b}{\varepsilon_0 A}$

4.)



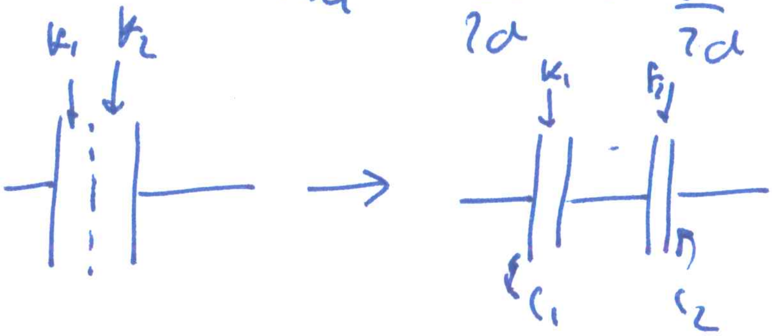
1.)




capacitance: $C_{eq} = C_1 + C_2$

$$\Rightarrow C_{eq} = \frac{\varepsilon_1 A}{2d} + \frac{\varepsilon_2 A}{2d} = (K_1 + K_2)\frac{\varepsilon_0 A}{2d}$$

2.)

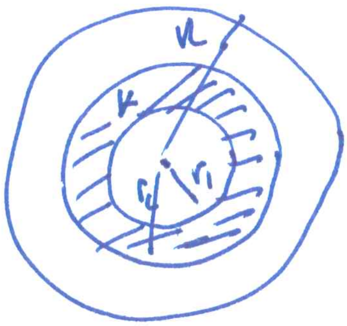



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/2}{\varepsilon_1 A} + \frac{d/2}{\varepsilon_2 A}$$

$$= \frac{d}{2A\varepsilon_0}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$C_{eq} = \frac{2A\varepsilon_0}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

5.)




capacitance per unit length

if line ch charge has $\lambda$

~~$\lambda$ polariz~~

$\lambda$ polarizes the dielectric cylinder

?

- outside dielectric: apply Gauss' law

$$E_r 2\pi r L = \frac{\lambda L}{\varepsilon_0}$$

$$\Rightarrow E_r = \frac{\lambda}{2\pi\varepsilon_0 r}$$

- inside dielectric $\quad E_r = \dfrac{\lambda}{2\pi\varepsilon r} = \dfrac{\lambda}{\kappa 2\pi\varepsilon_0 r}$

- potential difference

$$\Delta V = -\int_0^R E_r\,dr = -\int_0^{r_1} E_r\,dr - \int_{r_1}^{r_2} E_r\,dr - \int_{r_2}^{R} E_r\,dr$$

$$= -\frac{\lambda}{2\pi\varepsilon_0}\left[\ln(R) - \ln(0^+)\right] - \frac{\lambda}{2\pi\varepsilon}\left[\ln(r_2) - \ln(r_1)\right]$$

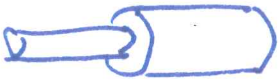
$$- \frac{\lambda}{2\pi\varepsilon_0}\left[\ln R - \ln r_2\right]$$

- diverges! helping assumption: line of charge has small, but finite thickness $R = 0^+$

$$C = \frac{1}{\frac{1}{2\pi\varepsilon_0}\left(\ln(R) - \ln(0^+)\right) + \frac{1}{2\pi\varepsilon}\left(\ln(r_2) - \ln(r_1)\right) + \frac{1}{2\pi\varepsilon_0}\left(\ln R - \ln r_2\right)}$$

6.) 1.) $I = 2.0\,A$

current density



$A_1 = \pi(.0012)^2$

$A_2 = \pi(.0025)^2$

$J_1 = I/A_1 \quad , \quad J_2 = I/A_2$ | $J_1 = 442097\ A/m^2$

$J_2 = 101859\ A/m^2$

2.) drift speed: $J = nq v_d$

$q = e$ (electrons) $1.6\times10^{-19}\,C$

n - number of charge carriers per volume

n: valence electrons / volume

density of copper: $8930\ kg/m^3 = \rho_{cu}$

density of aluminium: $2800\ kg/m^3 = \rho_{Al}$

number of valence electrons of copper: 1

number of valence electrons of aluminium: 3

$n_{cu} = \dfrac{\rho_{cu}}{m_{cu}} \times 1$ $\qquad$ $n_{Al} = \dfrac{\rho_{Al}}{m_{Al}} \times 3$

$m_{cu} = \dfrac{.063\ kg}{6\times10^{23}}$ $\qquad$ $m_{Al} = \dfrac{.027\ kg}{6\times10^{23}}$

$n_{cu} = \dfrac{6\times10^{23}}{.063} \times 8930$ $\qquad$ $n_{Al} = \dfrac{18\times10^{23}\times2800}{.027}$

$n_{cu} = 8.5\times10^{28}$ $\qquad$ $n_{Al} = 18\times10^{28}$

$\cancel{J_{cu}}\ v_{d,cu} = \dfrac{\cancel{442097}}{} 3.28\times10^{-5}\ m/s$ $\quad v_{d,Al} = 3.5\times10^{-6}\ m/s$

7.) $E = \rho J$

$V = Ed = \rho J d \qquad J = \frac{I}{A}$

$V = \frac{\rho I d}{A} = RI \qquad R = \frac{\rho d}{A}$

8.) $\rho_{Ag} = 1.62\times10^{-8}\ \Omega m$

$\rho = \frac{m}{ne^2\tau} \Rightarrow \tau = \frac{m}{ne^2\rho}$

$m =$ mass of $e^-$

$e = 1.6\times10^{-19}\ C$

$n_{al} =$ number of charge carriers/unit volume

$n_{Al} =$ (# of valence $e^-$'s/atom) (# of atoms/unit volume)

$n_{al} = 5.8\times10^{28}\ /m^3$

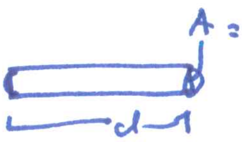
$g = \tau = 3.78\times10^{-14}$ seconds

9.) $J = J_0(1-(r/R)^2)$

$I = \int_0^R\int_0^{2\pi} r\,dr\,d\theta\,(1-(r/R)^2)$

$= 2\pi\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right] = 2\pi R^2\left(\frac{1}{4}\right)$

10.) $I = 300\,A \qquad \rho = 1.69\times10^{-8}\ \Omega m$




$A = 0.2\,cm^2 = 0.00002\ m^2$

$d = 0.85\,m$

$J = \frac{300}{.000021} \frac{A}{m^2} = 15000000$

$v_d = \frac{J}{n e}$ $\quad n = 8.5 \times 10^{28}$ (copper see problem 6)

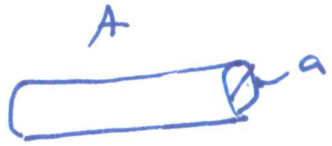
$e = 1.6 \times 10^{-19}$ C

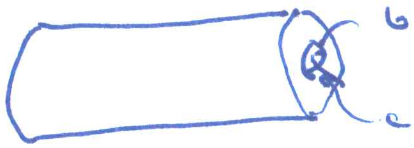
$v_d = .001102942 \frac{m}{s}$

time to pass .85m $\Rightarrow$ $t = \frac{.85}{.001102942}$ s ~ 771s

$=$

11.)




$\rho \rightarrow R_A = \frac{\rho L}{A} = \frac{4 \rho L}{\pi a^2}$ $\quad R_B = \frac{4 \rho L}{4\pi(c^2 - b^2)}$

$\frac{R_A}{R_B} = \frac{(c^2 - b^2)}{a^2}$

12.) 1.) $R_{eq} = R_1 + R_2 + R_3$

$\Delta V_1 = I R_1$

$\Delta V_2 = I R_2$

$\Delta V_3 = I R_3$

2.) $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 R_2 R_3}$

$R_{eq} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$

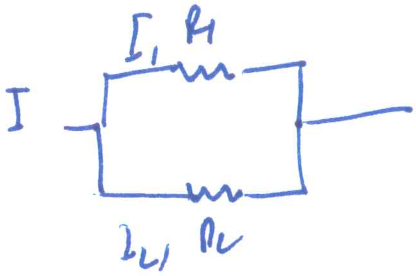
potential change is the same across each resistor

$$R_{TOT} \rightarrow \frac{R_1 R_2}{R_1 + R_2} + \frac{R_3 R_4}{R_3 + R_4} = \frac{100 \cdot 250}{350} + \frac{350 \cdot 200}{550}$$

$$= \frac{100 \cdot 5}{7} + \frac{7 \cdot 200}{11}$$

$$= \frac{500}{7} + \frac{1400}{11} = 198.7\,\Omega$$

total current: $I = \frac{24\,V}{198.7\,\Omega} = .12\,A$




$$I = I_1 + I_2$$

$$I_1 R_1 = I_2 R_2$$

$$\Rightarrow (I - I_2) R_1 = I_2 R_2$$

$$I R_1 = I_2 (R_1 + R_2)$$

$$I_2 = \frac{R_1}{R_1 + R_2}$$

$$I_1 = \frac{R_2}{R_1 + R_2}$$

current across $100\,\Omega$ resistor: $\frac{250}{350} I = \frac{5}{7} I$

$250\,\Omega$ resistor: $\frac{100}{350} I = \frac{2}{7} I$

$350\,\Omega$ resistor: $\frac{200}{550} \cancel{\frac{300}{550}} I = \frac{4}{11} I$

$200\,\Omega$ resistor: $\cancel{200}\ \frac{350}{550} I = \frac{7}{11} I$

calculate currents in terms of total current $I$

$\Delta V = I_1 R_1$

$\Delta V = I_2 R_2$

$\Delta V = I_3 R_3$

$I = I_1 + I_2 + I_3$

$I_1 R_1 = I_2 R_2$

$I_1 R_1 = I_3 R_3$

$I_2 R_2 = I_3 R_3$

$(I - I_2 - I_3) R_1 = I_2 R_2$

$I R_1 = I_2 (R_1 + R_2) + I_3 R_1$

$I_2 R_2 = I_3 R_3$

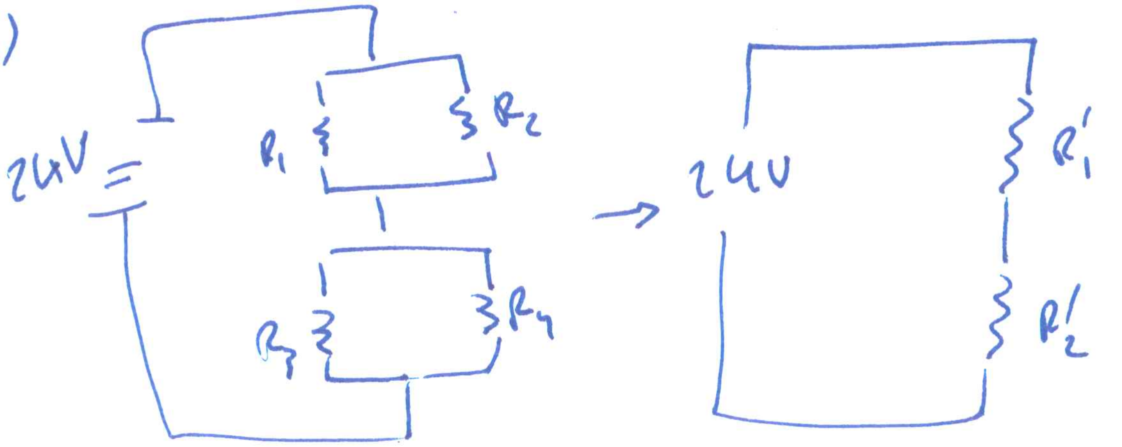
$I R_1 R_2 = I_3 (R_1 R_3 + R_1 R_2 + R_2 R_3)$

$$I_3 = \frac{R_1 R_2}{(R_1 R_2 + R_1 R_3 + R_2 R_3)} I$$

$$I_2 = \frac{R_1 R_3}{(R_1 R_2 + R_1 R_3 + R_2 R_3)} I$$

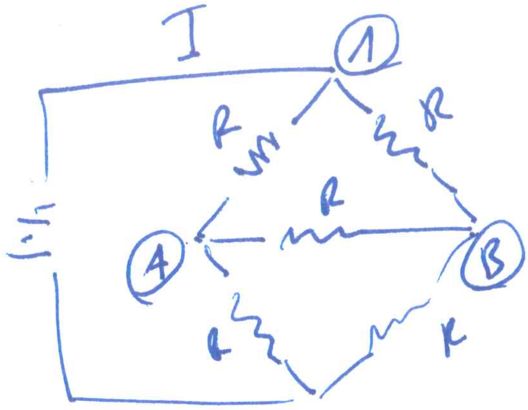
$$I_1 = \frac{R_2 R_3}{(R_1 R_2 + R_1 R_3 + R_2 R_3)} I$$

3.)




$$R_1' = \frac{R_1 R_2}{R_1 + R_2} \qquad R_2' = \frac{R_3 R_4}{R_3 + R_4}$$

13.

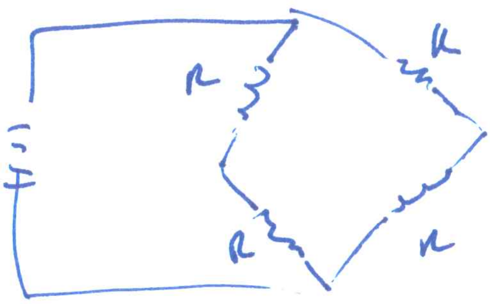


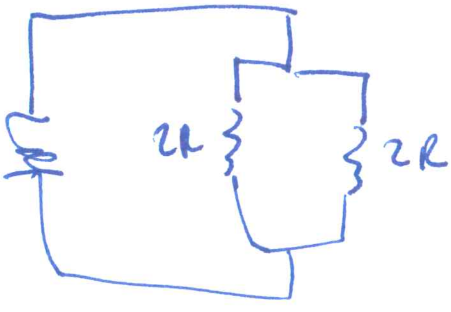

current divides evenly at 1

⇓

between A and B no potential drop

⇓




$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} = \frac{4R}{4R^2} = \frac{1}{R}$$

$$\boxed{R_{eq} = R}$$