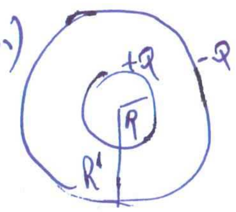
Problem Set-4

1.) a.)



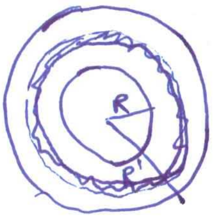
From Gauss's law $E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi r^2 \varepsilon_0}$

$$V = \frac{-Q}{4\pi\varepsilon_0}\int_{R'}^{R} dr \cdot \frac{1}{r^2} = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{-1}{r}\right)\Bigg|_{R'}^{R} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{R'}\right)$$

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 \left(\frac{1}{R} - \frac{1}{R'}\right)^{-1} = \frac{4\pi\varepsilon_0 R R'}{R' - R}$$

i) $C' = 4\pi\varepsilon_0 \frac{2R \cdot 2R'}{\cancel{2}(R'-R)} = 2 \cdot C$ → capacitance is doubled.

ii)



* If it has (finite) thickness, since inside the conductor potential difference is zero and outside the conductor (but between the plates) potential difference is what we've found in the part a, total potential difference between plates will be decreased.

$C = \frac{Q \leftrightarrow (\text{constant})}{V \searrow} \Longrightarrow C \nearrow$

* If its thickness is infinitesimal, then nothing would change.

b.) $C > C'$

Suppose C' holds Q' under V': $Q' = C'V'$

If C holds $2Q'$: $2Q' = C \cdot V \Rightarrow V = \frac{2Q'}{C} \Rightarrow \frac{2C'V'}{C}$

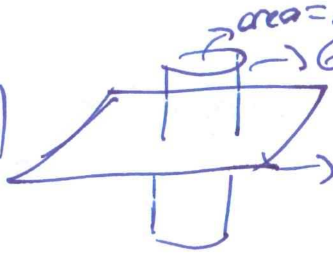
So V must be chosen as $\frac{2C'}{C} \cdot V'$.

c.) $C = \varepsilon \cdot \frac{A}{d}$
* increasing d.
* decreasing A.
* If it's not in the vacuum, you may decrease ε.

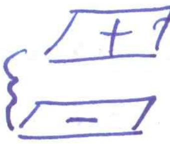
d.) So that capacitor would be discharged and wouldn't shock somebody in case of contact.

2.)

a.)



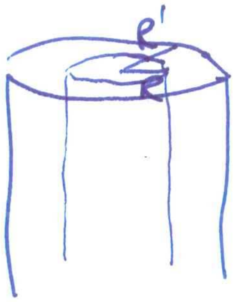
$$E \cdot 2A = \sigma \cdot \frac{A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$



$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow U = \frac{\sigma}{\varepsilon_0} \cdot d = \frac{Q \cdot A}{\varepsilon_0 d} \Rightarrow C = \varepsilon_0 \frac{A}{d}$$

b) Done.

c.)



Again from Gauss's law

$$E \cdot 2\pi r L = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi L \varepsilon_0} \cdot \frac{1}{r}$$

$$V = + \frac{Q}{2\pi L \varepsilon_0} \int_R^{R'} \frac{dr}{r} = \frac{+Q}{2\pi L \varepsilon_0} \ln\left(\frac{R'}{R}\right)$$

$$\Rightarrow C = \frac{Q}{V} = \frac{2\pi L \varepsilon_0}{\ln(R'/R)}$$

3.) $\varepsilon_0 = 8{,}85 \times 10^{-12} F/m$

a.) $C = \varepsilon_0 \dfrac{A}{d} = \dfrac{8{,}85 \cdot 10^{-12} \cdot \pi (8{,}8 \cdot 10^{-2})^2}{2 \cdot 10^{-3}}$ (I wrote all lengths in units of meter)

$\cong \pi \cdot 3{,}43 \times 10^{-11} F \quad \cong 1{,}08 \times 10^{-10} F$

b.) $Q = C \cdot V = 1{,}08 \times 10^{-10} \cdot 120 = 129{,}6 \times 10^{-10} \cong 1{,}30 \times 10^{-8}$ coulomb

4.) $U = \dfrac{Q^2}{2C}$, $C = \varepsilon_0 \cdot \dfrac{A}{x}$ $\quad U_1 = \dfrac{Q^2}{2} \cdot \dfrac{x}{\varepsilon_0 A}$, $U_2 = \dfrac{Q^2}{2} \dfrac{(x+dx)}{\varepsilon_0 A}$

$U_2 - U_1 = \dfrac{Q^2}{2\varepsilon_0 A} \cdot dx \Rightarrow F = \dfrac{Q^2}{2\varepsilon_0 A}$ $\quad (\Delta W = F \cdot \Delta x)$

5.) a.) We found the capacitance of a spherical capacitor.

$$C = 4\pi\varepsilon_0 \cdot \frac{R \cdot R'}{R' - R} = 4\pi\varepsilon \frac{R}{1 - \frac{R}{R'}} \xrightarrow[R' \to \infty]{} 4\pi\varepsilon_0 R$$

b.) This is also found.

$$C = \frac{2\pi L \varepsilon_0}{\ln(R'/R)} \xrightarrow[R' \to \infty]{} 0 \quad \left(\ln \frac{R'}{R} \xrightarrow[R' \to \infty]{} \infty\right)$$

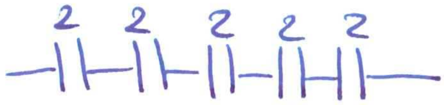
This means capacitor cannot hold charge.
Answer is no.

c-) $V_1 = \frac{4}{3}\pi R_1^3$

two drops form a single drop. $2V_1 = \frac{4}{3}\pi R_2^3 \Rightarrow R_2^3 = 2R_1^3 \Rightarrow R_2 = 2^{1/3} \cdot R_1$

$$\Rightarrow C = 4\pi\varepsilon_0 R_1 \cdot 2^{1/3}$$

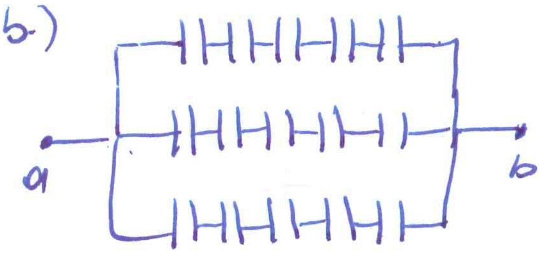
6-) a.)



→ series

$$5 \cdot \frac{1}{2} = \frac{1}{C} \Rightarrow C = \frac{2}{5} = 0{,}4\,\mu F$$

b.)



If you apply 1000 V between a and b, each capacitor feel 200 V between its plates. Since each capacitor is capable of withstanding 200 V, this is O.K.

$$C = 0{,}4 + 0{,}4 + 0{,}4 = 1{,}2\,\mu F$$

7-) $u = \frac{1}{2}\varepsilon_0 E^2$ → energy density

$E = \frac{Q}{2\pi L\varepsilon_0} \cdot \frac{1}{r}$ as found above.

$$U_0 = \text{Total Energy} = \frac{\varepsilon_0}{2} \cdot \frac{Q^2}{4\pi^2 L^2 \varepsilon_0^2} \int_a^b \int_0^{2\pi} \int_0^L \frac{1}{r^2} \cdot dz \cdot \overbrace{r \cdot d\theta \cdot dr}^{\text{area element}} = \frac{Q^2}{4\pi L \varepsilon_0} \cdot \ln\left(\frac{b}{a}\right)$$

$$U = \text{Energy (from } r=a \text{ to } r=\sqrt{ab}) = \frac{Q^2}{4\pi L\varepsilon_0} \cdot \underbrace{\ln\left(\frac{\sqrt{ab}}{a}\right)}_{= \ln\left(\sqrt{\frac{b}{a}}\right) = \frac{1}{2} \cdot \ln\left(\frac{b}{a}\right)} \Rightarrow \boxed{U = \frac{U_0}{2}}$$