olem Set 3, Solutions:



.) 1.) 1eV - work to carry one electron across a potential difference of 1V

1peV - same but electron → proton

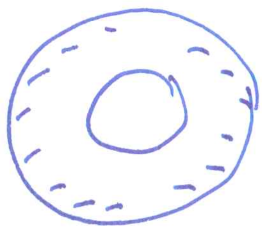
sign is different

2.) electrons move from low potential to high potential

3.) because negative charge exists, negative mass does not

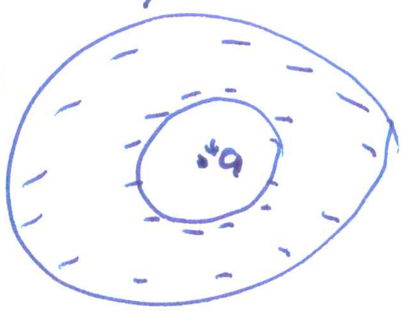
4.) constant

5.) spherical shell charge Q negative on outer surface



place charge q, positive, inside

1.) $|q| < |Q|$ ⇒

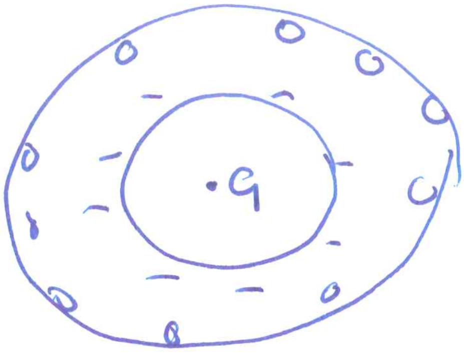



total charge on inner surface $-q$

total charge on outer surface $-(|Q| - |q|)$

1.) (con't)

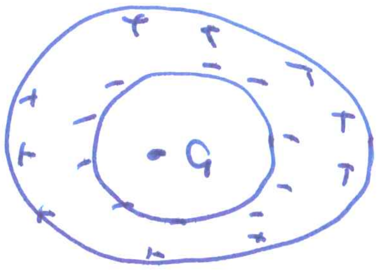
2.) $|Q| = |q|$




total charge on inner surface: $-q$

total charge on outer surface: $0$

3.) $|q| > |Q|$




total charge on inner surface: $-q$

total charge on outer surface: $+(|q| - |Q|)$

2.) 1.) $V = 10^9 V$ $\quad Q = 30C$

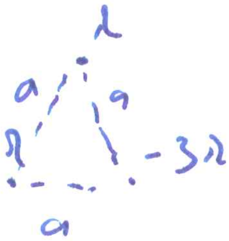
energy $\quad qV \Rightarrow 3 \cdot 10^{10} J$

2.) $\frac{mv^2}{2} = 3 \cdot 10^{10} J$ $\quad m = 1000 kg$

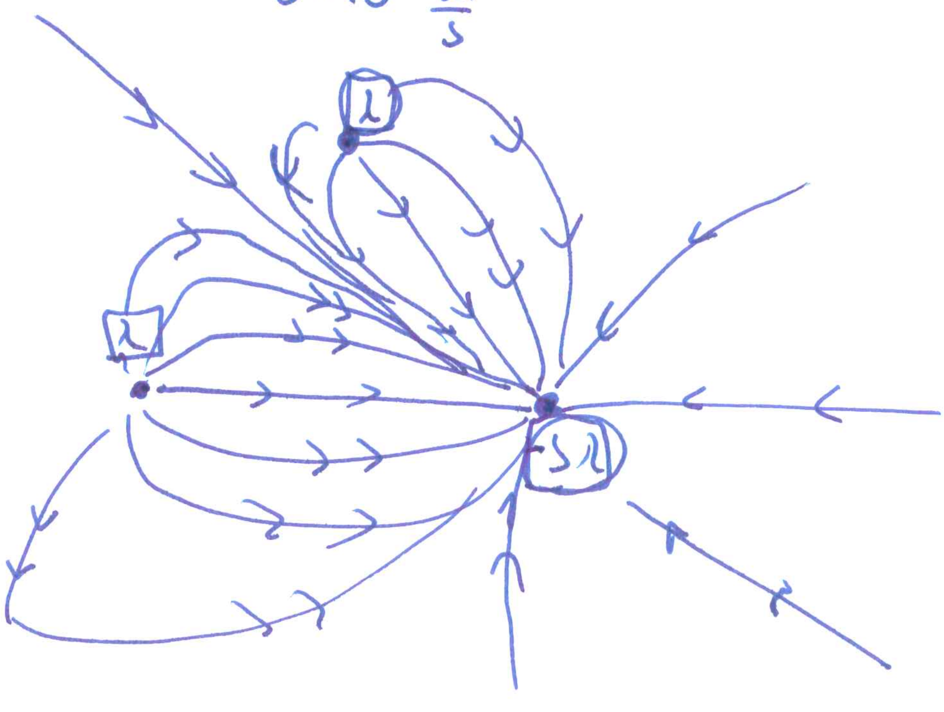
$v = ? \Rightarrow v^2 = 6 \cdot 10^7 \left(\frac{m}{s}\right)^2$

$v = 8 \cdot 10^3 \frac{m}{s}$

3.)




$\Rightarrow$



4.) 1.) potential difference

$$V - V' = \int_{r'\theta'\varphi'}^{r\theta\varphi} d\vec{r} \cdot \vec{\nabla} V(\vec{r})$$

$$= -\int_{P'}^{P} d\vec{r} \cdot \vec{E} = -\int_{P'}^{P} dr\, E_r$$

$$= -\int_{P'}^{P} dr'' \frac{Q}{4\pi\epsilon_0 r''^2}$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r''}\right]_{r'}^{r}$$

$$= \frac{Q}{4\pi\epsilon_0 r} - \frac{Q}{4\pi\epsilon_0 r'}$$

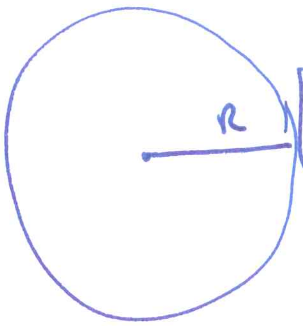
2.) line of charge

$$V - V' = \int_{P'}^{P} dr''\, E_{r''} \qquad E_{r''} = \frac{\lambda}{2\pi\epsilon_0 r''}$$

$$= \frac{\lambda}{2\pi\epsilon_0} \ln r \Big|_{r'}^{r} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r}{r'}$$

5.)

$\rho, R$




$\boxed{\text{case 1: } r, r' < R}$

$$V - V' = -\int_{r'}^{r} E_{r''}\, dr''$$

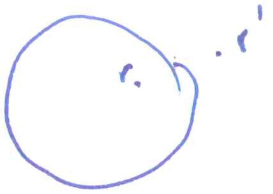
$$E_{r''} = ? \qquad E_r 4\pi r^2 = \frac{\rho 4\pi r^3}{3\epsilon_0}$$

$$E_r = \frac{\rho r}{3\epsilon_0}$$

$$V - V' = -\frac{\rho}{3\epsilon_0} \int_{r'}^{r} r''\, dr''$$

$$= -\frac{\rho}{3\epsilon_0} \frac{r''^2}{2}\Big|_{r'}^{r} = -\frac{\rho}{6\epsilon_0}\left(r^2 - r'^2\right)$$

$\boxed{\text{case 2: } r<R,\ r'>R}$



$$V - V' = -\int_{r'}^{R} E(r'')\,dr'' - \int_{R}^{r} E(r'')\,dr''$$

$$= -\int_{r'}^{R} \left(\frac{\rho}{3\epsilon_0}\right) r''\,dr'' - \int_{R}^{r} \frac{Q}{4\pi\epsilon_0 r''^2}\,dr''$$

$$= -\frac{\rho}{3\epsilon_0}\frac{r''^2}{2}\Big|_{r'}^{R} + \frac{Q}{4\pi\epsilon_0}\frac{1}{r''}\Big|_{R}^{r}$$

$$Q = \frac{4\pi}{3}R^3\rho$$

$$= -\frac{\rho}{6\epsilon_0}(R^2 - r^2) + \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r'} - \frac{1}{R}\right)$$

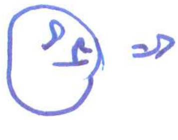
$\boxed{\text{case 3: } r, r' > R}$

$$V - V' = -\int_{r'}^{r} E(s)\,ds$$

$$= -\left(-\frac{Q}{4\pi\epsilon_0}\frac{1}{s}\right)_{r'}^{r}$$

$$= \frac{Q}{4\pi\epsilon_0 r} - \frac{Q}{4\pi\epsilon_0 r'}$$

6.) 1.)



$r < R$

$$V(r) = -\int_0^r ds\, E(s)$$

$$\cancel{= -\int_0^r ds \frac{Q}{4\pi\epsilon_0 s^2}}$$

$$\cancel{= -\int_0^r ds - \left[\frac{Q}{4\pi\epsilon_0 s}\right]_0^r}$$

$$V(r) = -\int_0^r ds\left(\frac{\rho}{3\epsilon_0}s\right) = -\frac{\rho}{6\epsilon_0}s^2\Big|_0^r = -\frac{\rho r^2}{6\epsilon_0}$$

$r > R$

$$V(r) = -\int_R^r E(s)\,ds + V(R)$$

$$= -\int_R^r E(s)\,ds - \frac{\rho R^2}{6\epsilon_0}$$

$$= -\int_R^r \frac{Q}{4\pi\epsilon_0 s^2}\,ds - \frac{\rho R^2}{6\epsilon_0}$$

$$Q = \frac{4\pi R^3}{3}\rho$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{s}\right]_R^r - \frac{\rho R^2}{6\epsilon_0}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right] - \frac{\rho R^2}{6\epsilon_0}$$

2.) $r > R$

$$V(r) = -\int_\infty^r E(s)\,ds = -\frac{Q}{4\pi\epsilon_0}\int_\infty^r \frac{ds}{s^2}$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{s}\right]_\infty^r = \frac{Q}{4\pi\epsilon_0 r}$$

$r < R$

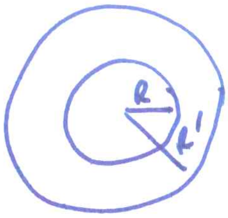
$$V(r) = -\int_R^r E(s)\,ds + V(R)$$

$$= -\int_R^r E(s)\,ds + \frac{Q}{4\pi\epsilon_0 R}$$

$$= -\int_R^r \frac{\rho}{3\epsilon_0} s\,ds + \frac{Q}{4\pi\epsilon_0 R}$$

$$= -\frac{\rho}{6\epsilon_0} s^2 \Big|_R^r + \frac{Q}{4\pi\epsilon_0 R}$$

$$= -\frac{\rho}{6\epsilon_0}(r^2 - R^2) + \frac{Q}{4\pi\epsilon_0 R}$$

7. 

take $V(r=\infty) = 0$

uniform charge density: $\rho$

6

total charge: $Q = \frac{4\pi\rho}{3}(R'^3 - R^3)$

potential outside $V(r) = \frac{Q}{4\pi\epsilon_0 r} = \frac{4\pi\rho}{3}\frac{(R'^3 - R^3)}{r}$

potential in region $R < r < R'$

Gauss' law: $E_r 4\pi r^2 = \frac{4\pi\rho(r^3 - R^3)}{3\epsilon_0}$

$$E_r = \frac{\rho}{3\epsilon_0}\left(r - \frac{R^3}{r^2}\right)$$

$$V(r) = -\int_{R'}^{r} E(s)\,ds$$

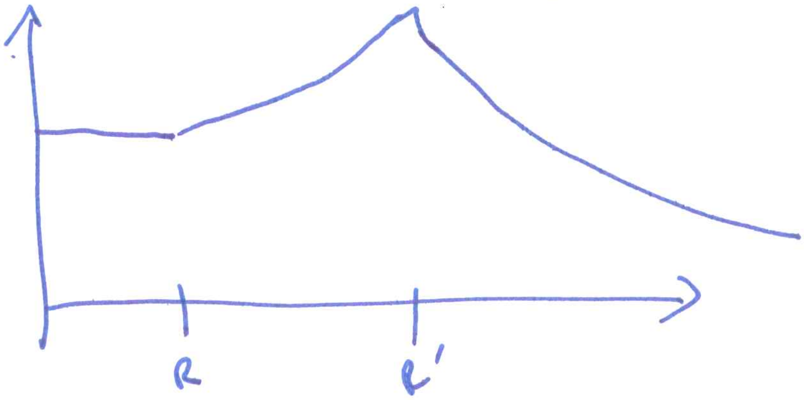
$$= -\int_{R'}^{r} ds\left(\frac{\rho}{3\epsilon_0}\right)\left(s - \frac{R^3}{s^2}\right)$$

$$= -\left(\frac{\rho}{3\epsilon_0}\right)\int_{R'}^{r} ds\left(s - \frac{R^3}{s^2}\right)$$

$$= -\left(\frac{\rho}{3\epsilon_0}\right)\left[\frac{s^2}{2} + \frac{R^3}{s}\right]_{R'}^{r}$$

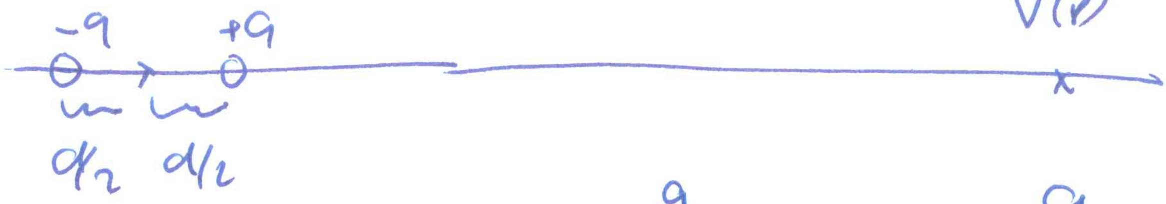
$$= -\frac{\rho}{3\epsilon_0}\left[\frac{r^2}{2} + \frac{R^3}{r} - \frac{R'^2}{2} - \frac{R^3}{R'}\right]$$

at $V(r=R) = -\frac{\rho}{3\epsilon_0}\left[\frac{R^2 - R'^2}{2}\right]$



7

8.)




$$V(x) = \frac{q}{4\pi\epsilon_0 (x - d/2)} - \frac{q}{4\pi\epsilon_0 (x + d/2)}$$

$$\cancel{\frac{1}{x+\delta}} = ? = \frac{1}{x} + \frac{\delta x}{x^2} + \dots$$

$$\frac{1}{x(1+\frac{\delta}{x})} = \frac{1}{x}\left(\frac{1}{1+\frac{\delta}{x}}\right)$$

expand: $\frac{1}{1+x}$

$$f(x) = \frac{1}{1+x}$$

$$f'(x) = -\frac{1}{(1+x)^2}$$

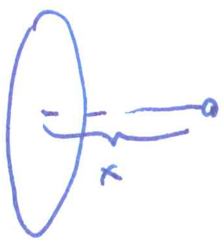
$$\cancel{f''(x) =}$$

$$V(x) = \frac{q}{4\pi\epsilon_0}\frac{1}{x}\left(1 + \frac{d}{2x}\right) - \frac{q}{4\pi\epsilon_0}\frac{1}{x}\left(1 - \frac{d}{2x}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\frac{d}{x^2} = \frac{p}{4\pi\epsilon_0 x^2} \qquad \boxed{p = qd}$$

9.) $\sigma(r) = A\frac{r}{R}$

potential:




ring: $$dV(x) = \frac{1\,\cancel{2\pi} r\,(A r/R)}{2\,\cancel{4\pi}\epsilon_0 (x^2 + r^2)^{1/2}}$$

$$= \frac{A}{2R\epsilon_0}\int_0^R \frac{dr\; r^2}{(x^2 + r^2)^{1/2}}$$

$$= \frac{A}{2R\epsilon_0}\int_0^R \frac{dr}{(1 + \frac{x^2}{r^2})^{1/2}}$$

do integral as indefinite:

$$\int_0^R \frac{dr}{\left(1+\frac{x^2}{r^2}\right)^{1/2}} \qquad u = \frac{x}{r} \rightarrow r = \frac{x}{u}$$

$$dr = -\frac{x}{u^2}\,du$$

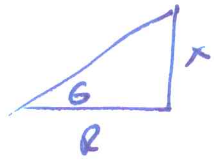
$$= -x \int_\infty^{x/R} \frac{du}{u^2(1+u^2)^{1/2}} \qquad \sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$u = \tan\theta \Rightarrow \frac{du}{d\theta} = 1 - \frac{\sin\theta}{\cos^2\theta}(-\sin\theta)$$

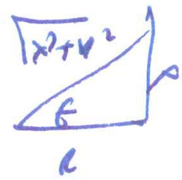
$$= 1 + \tan^2\theta$$

$$= -x \int_\infty^{x/R} \frac{(1+\tan^2\theta)^{1/2}\,d\theta}{\tan^2\theta} \qquad \tan\theta \Rightarrow$$




$$= -x \int_\infty^{x/R} \frac{\sec\theta\,d\theta\,\cos^2\theta}{\sin^2\theta}$$

$$= -x \int_\infty^{x/R} \frac{\cos\theta\,d\theta}{\sin^2\theta} \qquad z = \sin\theta$$




$$\frac{dz}{d\theta} = \cos\theta$$

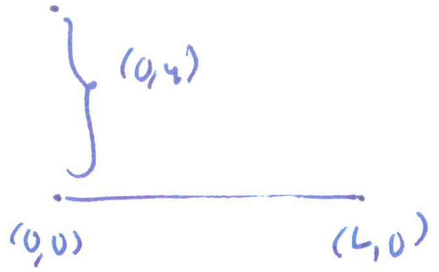
$$= -x \int \frac{dz}{z^2}$$

$$= \left.\frac{x}{z}\right|_1^{\frac{x}{\sqrt{x^2+R^2}}} = \sqrt{x^2+R^2} - x$$

$$\boxed{V(x) = \frac{A}{2R\varepsilon_0}\left[\sqrt{x^2+R^2} - x\right]}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{A}{2R\varepsilon_0}\left[\frac{2x}{2\sqrt{x^2+R^2}} - 1\right]$$

10.




for homogeneous charge density: $\lambda$

$$V(0,y) = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda\, dx}{\sqrt{x^2+y^2}}$$

$$= \frac{\lambda}{4\pi\epsilon_0 \cancel{y}}\int_0^{L/y} \frac{dx}{\sqrt{1+\frac{x^2}{y^2}}} \qquad \tilde{x} = \frac{x}{y}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{d\tilde{x}}{\sqrt{1+\tilde{x}^2}} \qquad \text{identity: } \cosh^2\theta = 1 + \sinh^2\theta$$

substitution:

$$\tilde{x} = \sinh\theta$$

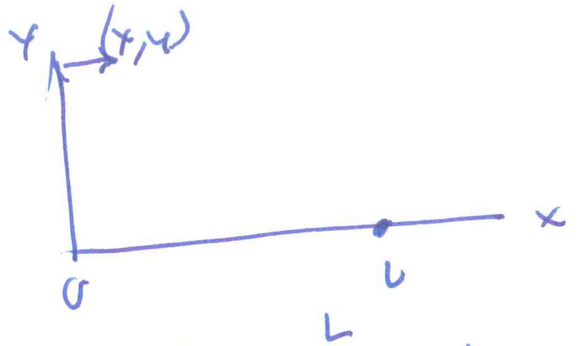
$$\frac{d\tilde{x}}{d\theta} = \cosh\theta$$

$$= \frac{\lambda}{4\pi\epsilon_0}\int_0^{\operatorname{asinh} L/y} \frac{\cancel{\cosh\theta}\, d\theta}{\cancel{\cosh\theta}}$$

$$= \frac{\lambda}{4\pi\epsilon_0} \operatorname{asinh}\left(\frac{L}{y}\right)$$

2.) can not determine E-field by simply ~~integrating~~ taking derivative since x-dependence was eliminated by setting $x = 0$

Solution: Solve using finite x




$$V(x,y) = \frac{\lambda}{4\pi\epsilon_0}\int_0^L \frac{dx'}{\sqrt{(x-x')^2+y^2}} \qquad \tilde{x} = x' - x$$

$$= \frac{\lambda}{4\pi\epsilon_0}\int_{-x}^{L-x} \frac{d\tilde{x}}{\sqrt{\tilde{x}^2+y^2}}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\int_{-x/y}^{(L-x)/y} \frac{d\tilde{\tilde{x}}}{\sqrt{1+\tilde{\tilde{x}}^2}} \qquad \tilde{\tilde{x}} = \sinh\theta \qquad \frac{d\tilde{\tilde{x}}}{d\theta} = \cosh\theta$$

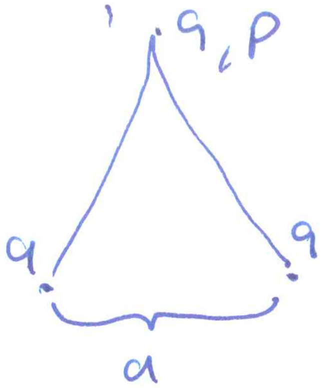
$$= \frac{\lambda}{4\pi\epsilon_0}\left[\operatorname{asinh}\left[\left(\frac{L-x}{y}\right)\right] - \operatorname{asinh}\left[-\frac{x}{y}\right]\right]$$

$$\cancel{= \frac{\lambda}{}}$$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{\cosh(\frac{L-x}{y})}\left(-\frac{1}{y}\right) - \frac{1}{\cosh(\frac{-x}{y})}\left(-\frac{1}{y}\right)\right]$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\lambda}{4\pi\epsilon_0}\left(\frac{1}{\cosh(\frac{L-x}{y})}\left(+\frac{x}{y^2}\right) - \frac{1}{\cosh(\frac{-x}{y})}\left(\frac{x}{y^2}\right)\right)$$

11.)

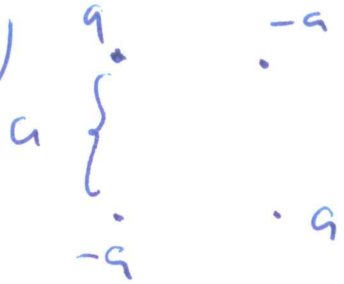



potential charge

$q(V(q \text{ at } P) - V(q \text{ at } \infty))$

$= \frac{q^2}{4\pi\epsilon_0 d} + \frac{q^2}{4\pi\epsilon_0 d}$

12.)



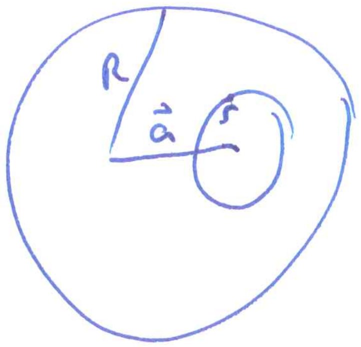

potential energy:

$2\frac{q^2}{4\pi\epsilon_0 (2a^2)}$

$-\cancel{4}\frac{q^2}{4\pi\epsilon_0 a^2}$

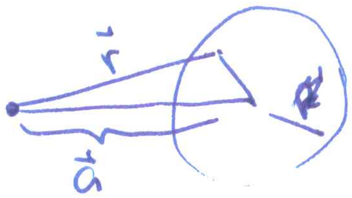
$= -\frac{3q^2}{4\pi\epsilon_0 a^2}$

13.) $\rho$, $R$

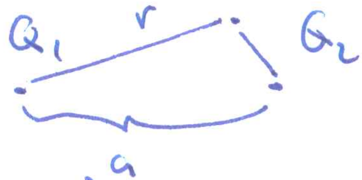



potential inside cavity

- consider some pt. $\vec{r}$
- can be considered as two point charges: one representing sphere, one the cavity




$\Rightarrow$




$Q_1 = \frac{4\pi\rho R^3}{3}$

$Q_2 = -\frac{4\pi\rho}{3}|\vec{r}-\vec{a}|^3$

12

potential:

$$V(r) = \frac{\cancel{4\pi}\rho}{3\epsilon_0} r^2 - \frac{\rho}{3\epsilon_0} |\vec{r} - \vec{a}|^2$$

field inside cavity: need $\frac{\partial}{\partial x}\sqrt{(x-a_x)^2+(y-a_y)^2+(z-a_z)^2}$

$$E_x = -\frac{\partial V}{\partial x} = \frac{\rho\, 2x}{3\epsilon_0}\frac{x}{x} - \frac{\rho}{3\epsilon_0} 2(x-a_x) = \frac{2\rho}{3\epsilon_0} \vec{a}_x$$

$$E_y = -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = \frac{2\rho}{3\epsilon_0} \vec{a}$$