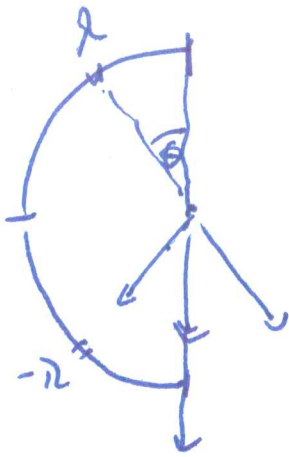


1.)



$$dQ = \lambda R d\theta$$

E-field points in direction  $\downarrow$



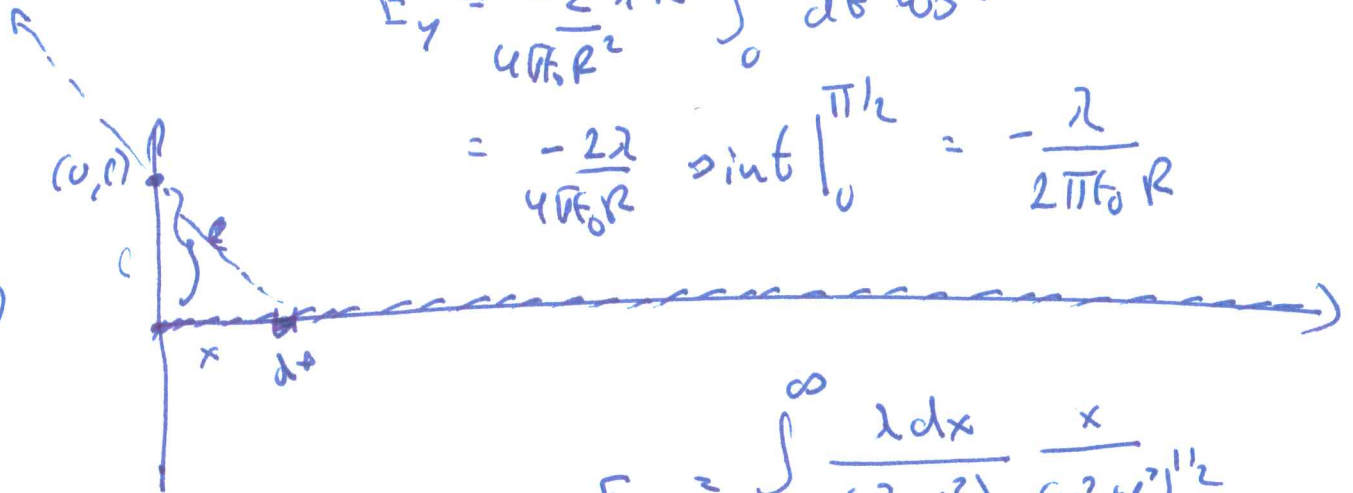
$$0 < \theta < \pi/2$$

$$E_y = - \int_0^{\pi/2} \frac{dQ}{R^2} \cos \theta \left( \frac{1}{4\pi\epsilon_0} \right)$$

$$E_y = - \frac{2\lambda R}{4\pi\epsilon_0 R^2} \int_0^{\pi/2} d\theta \cos \theta$$

$$= - \frac{2\lambda}{4\pi\epsilon_0 R} \sin \theta \Big|_0^{\pi/2} = - \frac{\lambda}{2\pi\epsilon_0 R}$$

2.)



Field at (0, l)

$$E_x = \int_0^{\infty} \frac{\lambda dx}{(x^2 + l^2)} \frac{x}{(x^2 + l^2)^{3/2}}$$

$$E_x = \lambda \int_0^{\infty} \frac{x dx}{(x^2 + l^2)^{3/2}}$$

$$\Rightarrow u = x^2 + l^2$$

$$du = 2x dx$$

$$= \frac{\lambda}{2} \int_l^{\infty} \frac{du}{u^{3/2}} = \frac{\lambda}{2} \left( -\frac{2}{u^{1/2}} \right)_l^{\infty}$$

$$= - \frac{\lambda}{l}$$

$$E_y = \lambda \int_0^{\infty} \frac{dx}{(x^2 + l^2)^{3/2}} = \frac{\lambda}{l} \int_0^{\infty} \frac{dx'}{(x'^2 + 1)^{3/2}}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

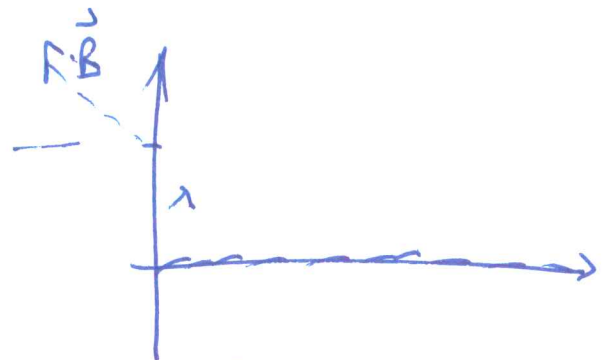
$$x' = \tan \theta$$

$$\frac{dx'}{d\theta} = \frac{d \tan \theta}{d\theta \cos \theta} = 1 + \tan^2 \theta$$

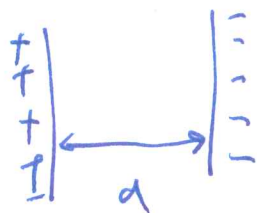
$$\int \frac{dx'}{(1+x'^2)^{3/2}} = \int \frac{(1+\tan^2 \theta) d\theta}{(1+\tan^2 \theta)^{3/2}}$$

$$= \int_0^{\pi/2} \frac{d\theta}{\sec \theta} = \int_0^{\pi/2} \cos \theta d\theta = -1$$

$$F_y = -\frac{\lambda}{c} \Rightarrow$$



3.)



$$d = 2.5 \text{ cm} = .025 \text{ m}$$

$$t = 2.2 \times 10^{-7} \text{ s}$$

acceleration is uniform:  $v_0 = 0$   $a = \text{constant}$   
 $x_0 = 0$

$$v = at$$

$$x = \frac{at^2}{2} \Rightarrow \text{when } x = .025 \text{ m}$$

$$t = 2.2 \times 10^{-7} \text{ s}$$

$$\frac{2(.025)}{(2.2 \times 10^{-7})^2} = a$$

$$\frac{2.2 \times 2.2}{444}$$

$$a = \frac{.05}{4.84} \times 10^{14}$$

$$= \frac{5.00}{4.84} \times 10^{12} \Rightarrow$$

$$5.00 \div 4.84 = 1.03$$

$$a = 1.03 \times 10^{12} \text{ m/s}^2$$

electric field = ?

$$F = q_e E = m_e a$$

$$\Rightarrow E = \frac{m_e a}{q_e} = \frac{9.11 \times 10^{-31} \text{ kg} \times 1.05 \times 10^{16} \text{ m/s}^2}{1.60 \times 10^{-19} \text{ C}}$$

$$= \frac{9.11 \times 1.05}{1.60}$$

$$\frac{9.11 \times 1.05}{1.60}$$

$$\frac{9.11 \times 1.05}{1.60} = \frac{9.5833}{1.60}$$

$$9.5833 \div 1.6000 = 5.81$$

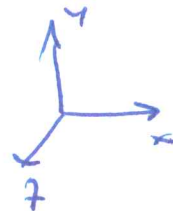
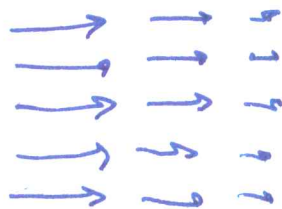
$$E = 5.81 \frac{\text{N}}{\text{C}} \quad \text{or} \quad \frac{\text{V}}{\text{m}}$$

4.

$$|\vec{v}| \propto R^2$$

5.

$\vec{E}$ -field



symmetry: charge density is constant in  
z and y directions

decreases in x-direction

force lines are same as  $\vec{E}$ -field lines

6.) Gauss law for gravitation

- for electrostatics we have

$$\text{Coulomb's law } f = \frac{qQ}{4\pi\epsilon_0 r^2}$$

$$\vec{E}\text{-field of } Q \Rightarrow |\vec{E}| = \frac{Q}{4\pi\epsilon_0 r^2}$$

$\Downarrow$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

Newton's law of gravitation:

$$F = G \frac{mM}{r^2}$$

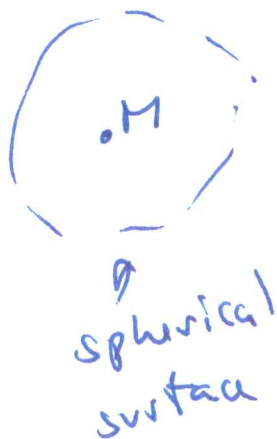
gravitational field  $\vec{g}$  :

$$|\vec{g}| = \frac{GM}{r^2}$$

$$\oint \vec{g} \cdot d\vec{A} = 4\pi GM$$

$\rightarrow$  does it reproduce Newton's law?

$$\oint \vec{g} \cdot d\vec{A} = 4\pi GM$$



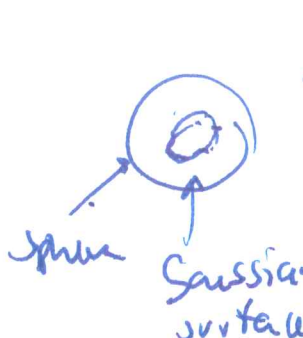
$$\Rightarrow \text{field} \Rightarrow g_r 4\pi r^2 = 4\pi GM$$
$$\Rightarrow g_r = \frac{GM}{r^2}$$

$\Downarrow$

Newton's law

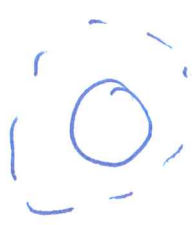
$$F_g = mg_r = \frac{GmM}{r^2}$$

11.1


 sphere  
 Gaussian surface

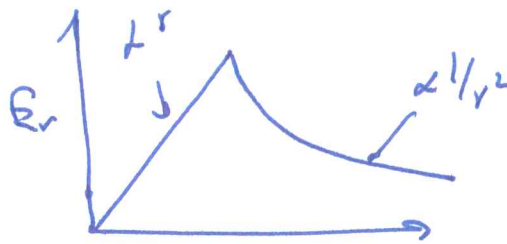
$\vec{E}$ -field for  $r < R$   
 $\oint \vec{E} \cdot d\vec{A} = E_r 4\pi r^2 = \frac{Q_{enclosed}}{\epsilon_0} = \frac{4\pi r^3 \rho}{3\epsilon_0}$

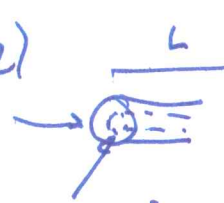
$\Rightarrow E_r = \frac{\rho}{3\epsilon_0} r$

$r > R$   

 sketch:

$\oint \vec{E} \cdot d\vec{A} = E_r 4\pi r^2 = \frac{Q}{\epsilon_0} = \frac{4\pi R^3 \rho}{3\epsilon_0}$

$\Rightarrow E_r = \frac{\rho R^3}{3\epsilon_0 r^2}$



11.2. cylinder (R)  

 charge density  $\rho$   
 Gaussian surface (r)

$r < R$

$$\oint \vec{E} \cdot d\vec{A} = E_r L 2\pi r$$

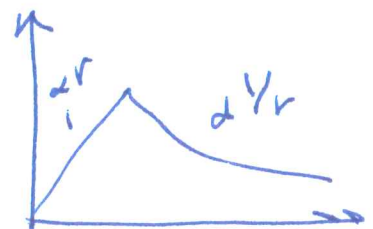
$$\frac{Q_{enclosed}}{\epsilon_0} \Rightarrow \frac{\rho \pi r^2 L}{\epsilon_0} = E_r L 2\pi r$$

$$E_r = \frac{\rho}{2\epsilon_0} r$$

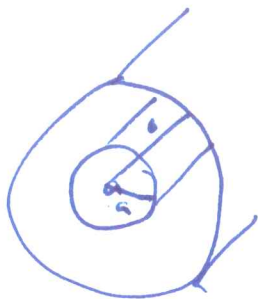
$r > R$

$$\frac{\rho \pi R^2 L}{\epsilon_0} = E_r L 2\pi r$$

$$E_r = \frac{\rho}{2\epsilon_0} \frac{R^2}{r}$$



8.)



$$a < b$$

$$r < a$$

$$E_r = 0$$

$$r > b$$

$$E_r = 0$$

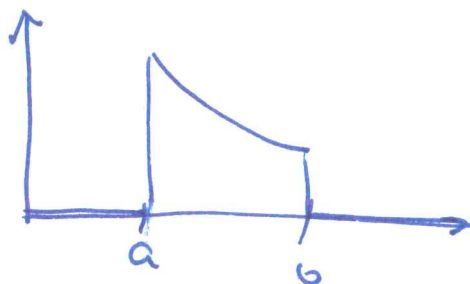
for  $a < r < b$

Gauss' law:

$$\oint \vec{E} \cdot d\vec{l} = \frac{Q_{enc}}{\epsilon_0}$$

$$E_r \cdot 2\pi r L = \frac{\sigma 2\pi a L}{\epsilon_0}$$

$$E_r = \frac{\sigma a}{\epsilon_0 r}$$



9.)

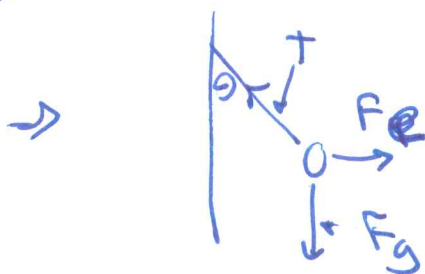


given:  $m, q, E$

forces on sphere:

1. gravity
2. electrostatic
3. tension

coord. system:



$$F_x = T - F_g \cos \theta - F_e \sin \theta = 0$$

$$F_x = F_g \sin \theta - F_e \cos \theta = 0$$

$$\Downarrow$$

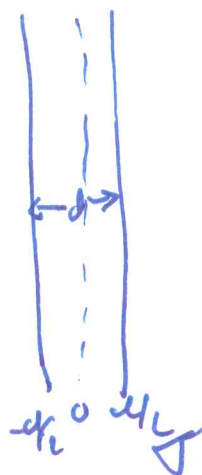
$$mg \tan \theta = qE$$

electric field is constant  $\frac{\sigma}{2\epsilon_0}$

$$mg \tan \theta = \frac{q\sigma}{2\epsilon_0}$$

$$\sigma = \frac{2\epsilon_0 mg \tan \theta}{q}$$

(6.)



$\rho$  - charge density

coordinate system: centered on at  
halfway (median plane)

E-field inside from Gauss' law

$$|E_x| \times 2A = \frac{\rho A (2x)}{\epsilon_0}$$

$$\Rightarrow |E_x| = \frac{\rho x}{\epsilon_0}$$

outside  $|E_x| \times \lambda A = \frac{\rho A \lambda d}{\epsilon_0}$

$$|E_x| = \frac{\rho d}{\epsilon_0}$$

(11.)

$$\rho(r) = \rho_0 r / R$$

$r < R$  E-field inside

$$E_r \pi r^2 =$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$Q_{\text{enclosed}} = \int_0^r \rho(r') r' dr' = \int_0^r \rho_0 \frac{r'}{R} r' dr' = \frac{\rho_0}{R} \int_0^r r'^2 dr' = \frac{\rho_0}{R} \left[ \frac{r'^3}{3} \right]_0^r = \frac{\rho_0 r^3}{3R}$$

$$Q_{\text{enclosed}} = \int_0^r (4\pi r') \left( \frac{\rho_0 r'}{R} \right) r' dr' = \frac{4\pi \rho_0}{R} \int_0^r r'^3 dr' = \frac{4\pi \rho_0}{R} \left[ \frac{r'^4}{4} \right]_0^r = \frac{\pi \rho_0 r^4}{R}$$

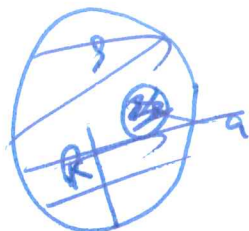
$$E_r 4\pi r^2 = \frac{\pi \epsilon_0 r^4}{\epsilon_0 R} \Rightarrow E_r = \frac{\epsilon_0 r^2}{4\epsilon_0 R}$$

outside:

Enclosed becomes  $\pi \epsilon_0$

$$E_r = \frac{\cancel{\pi \epsilon_0}}{4\cancel{\pi \epsilon_0} R r^2} \quad \frac{\pi \epsilon_0}{\epsilon_0 (4\pi r^2)} = \frac{\epsilon_0}{4\pi r^2}$$

12.)

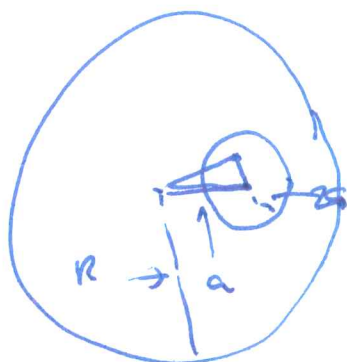


$$(a) \oint \vec{E} \cdot d\vec{f} = E_r 4\pi r^2 = \frac{Q_{\text{enclosed}}}{\epsilon_0} = \frac{q \frac{4\pi r^3}{3}}{3\epsilon_0}$$

$$\Rightarrow E_r = \frac{q}{3\epsilon_0} r$$

direction radial  $\rightarrow E_r = \frac{q}{3\epsilon_0} \vec{r}$

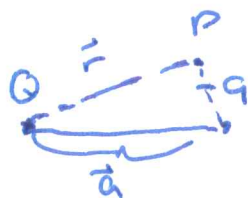
(b)



field inside cavity  
can be taken as sum of  
+ charge and - charge

$Q$  = enclosed within  
radius  $|\vec{r}|$

$q$  = enclosed within radius  
 $|\vec{r} - \vec{a}|$



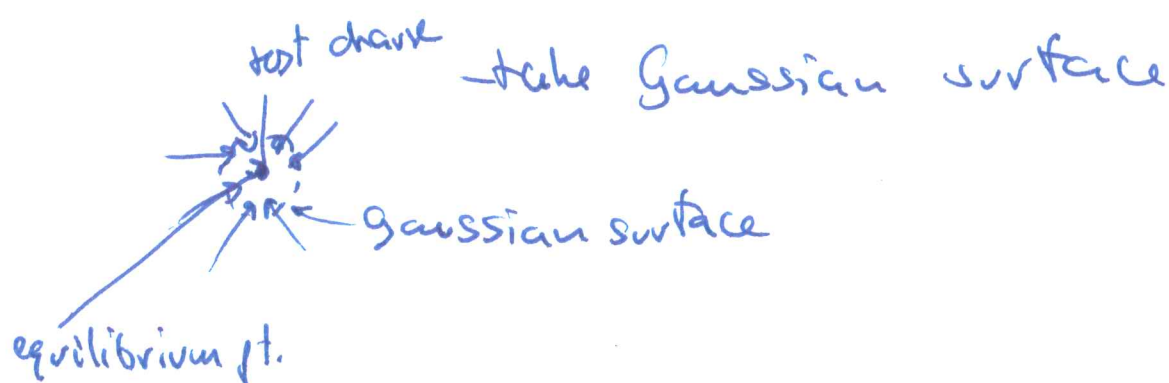
$$Q = \frac{4\pi}{3} r^3 q \quad q = -\frac{4\pi}{3} (r-a)^3 q$$

$$\vec{E} = \frac{q}{3\epsilon_0} \vec{r} + -\frac{q}{3\epsilon_0} (\vec{r} - \vec{a}) = \frac{q}{3\epsilon_0} \vec{a}$$

13.)

13.

for stable equilibrium forces point towards  
equilibrium point



evaluate flux over Gaussian surface

1.) contribution from test charges

2.) contribution from  $\vec{E}$ -field lines

contribution from 2. is finite and opposite  
to part 1.

finite contribution from 2  $\Rightarrow$  there is a  
charge inside Gaussian surface

$\Downarrow$   
would attract test charge

and the point considered would not  
be an equilibrium point