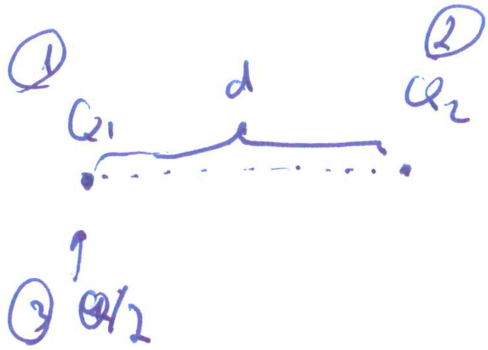
Problem Set 1: Solutions

1.)



1.) after 1st touch

$Q_1 = \frac{3Q}{4}$ $\quad Q_2 = Q$

$Q_3 = \frac{3Q}{4}$

2.) after second touch

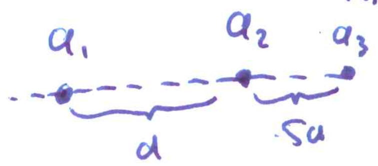
$Q_2 = \frac{7Q}{8}$ $\quad Q_3 = \frac{7Q}{8}$

$Q = \frac{3Q}{4}$

force: $F = \frac{1}{4\pi\epsilon_0}\left(\frac{3Q}{4}\right)\left(\frac{7Q}{8}\right)\frac{1}{d^2}$

$= \frac{1}{4\pi\epsilon_0}\frac{21Q^2}{32d^2} = \frac{21Q^2}{128\pi\epsilon_0 d^2}$

2.)



$F_{1\,on\,3} = \frac{q_1 q_3}{4\pi\epsilon_0 \left(\frac{3d}{2}\right)^2} = \frac{q_1 q_3}{9\pi\epsilon_0 d^2}$

$F_{2\,on\,3} = \frac{q_2 q_3}{4\pi\epsilon_0 \left(\frac{d}{2}\right)^2} = \frac{q_2 q_3}{\pi\epsilon_0 d^2}$

$F_{1\,on\,3} + F_{2\,on\,3} = 0$

$\Rightarrow \frac{q_1}{9} + q_2 = 0 \Rightarrow q_2 = -\frac{q_1}{9}$

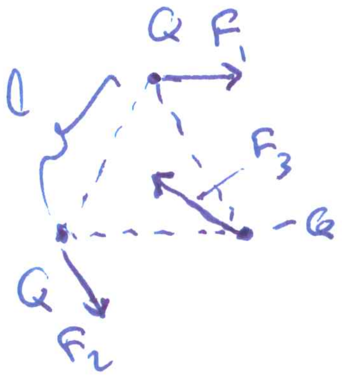
in between: $q_1 = q_2$

3.) 160 N/C

$$F_g = mg = 1kg\,(9.8)$$
$$= 9.8\,N$$

$$F_c = -F_g = q_0 E$$

$$q_0 = -\frac{9.8N}{160\,N/C} = -.06\,C$$

4.) 

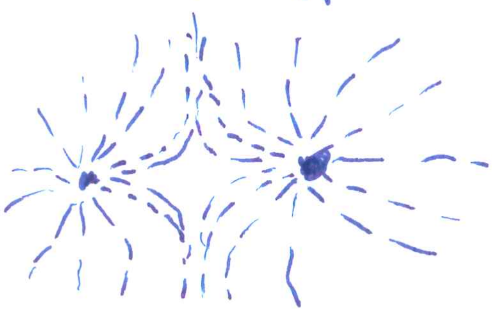
magnitudes:

$$F_1 = F_2 = \frac{Q^2}{4\pi\varepsilon_0 l^2}$$

$$F_3 = \frac{Q^2}{4\pi\varepsilon_0 l^2}\sqrt{3}$$

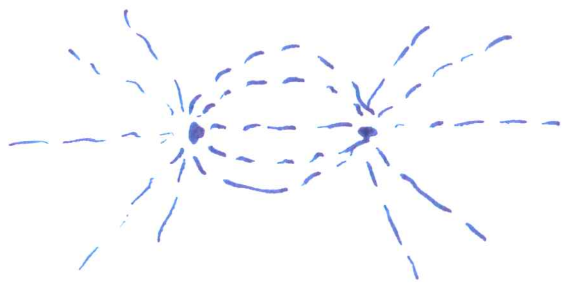
5.) One can determine whether charges are of the same sign or of opposite signs

same sign:
seeds line up as

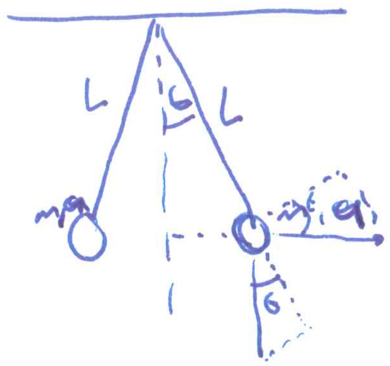


opposite signs:

seeds line up as



whether they are +, or − we can not determine (seeds do not have arrows)

6.) 

angle is small

tension in rope balances gravity + electrostatic force

forces perpendicular to rope balance as well

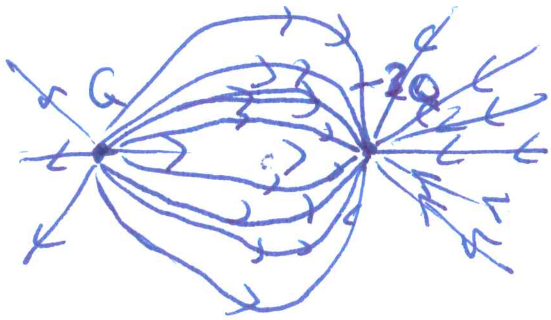
$$\frac{Q^2}{4\pi\epsilon_0 d^2}\cos\theta = mg\sin\theta$$

$$\Rightarrow \frac{Q^2}{4\pi\epsilon_0 d^2} = mg\tan\theta = mg\sin\theta$$

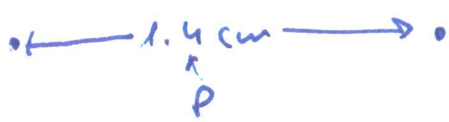
$$\sin\theta = \frac{d}{L}$$

$$\frac{Q^2}{4\pi\epsilon_0 d^2} = \frac{mg\,d}{L} \rightarrow d = \left(\frac{LQ^2}{4\pi\epsilon_0 mg}\right)^{1/3}$$

7.) 



8.) $Q = 1.4\times10^{-8}\,C$ $\quad Q = -1.4\times10^{-8}\,C$

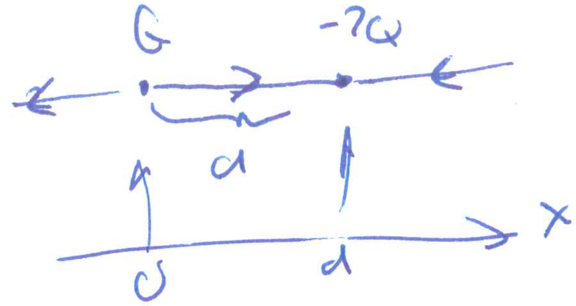


$$E(p) = \frac{Q}{4\pi\epsilon_0(.7\ cm)^2} + \frac{Q}{4\pi\epsilon_0(.7\,cm)^2}$$

$$= \frac{2Q}{4\pi\epsilon_0(.007)^2} = \frac{2(1.4\times10^{-8})}{(.007)^2}\times 8.988\times10^9$$

$$= 51.\times10^5\ \frac{V}{m}$$

9.)



(generally)

$x < 0$ $\quad E(x) = -\dfrac{Q}{4\pi\epsilon_0 x^2} + \dfrac{2Q}{4\pi\epsilon_0 (x+d)^2}$

$0 < x < d$ $\quad E(x) = \dfrac{Q}{4\pi\epsilon_0 x^2} + \dfrac{2Q}{4\pi\epsilon_0 (x-d)^2}$

$x > d$ $\quad E(x) = \dfrac{Q}{4\pi\epsilon_0 x^2} - \dfrac{2Q}{4\pi\epsilon_0 (x-d)^2}$

find zeroes: for $x < 0$

$$\frac{1}{x^2} = \frac{2}{(x+d)^2}$$

$$x^2 + 2xd + d^2 = 2x^2$$

$$x^2 - 2xd - d^2 = 0$$

$$x = \frac{2d \pm \sqrt{4d^2 + 4d^2}}{2} = d \pm \sqrt{2}d$$

only $d - \sqrt{2}d$ is in $x < 0$

$(1-\sqrt{2})d$

$x \to \infty$ is also zero

$x \to -\infty$ is also zero

for $x > d$

$$\frac{1}{x^2} = \frac{2}{(x-d)^2}$$

$$x^2 - 2xd + d^2 = 2x^2$$

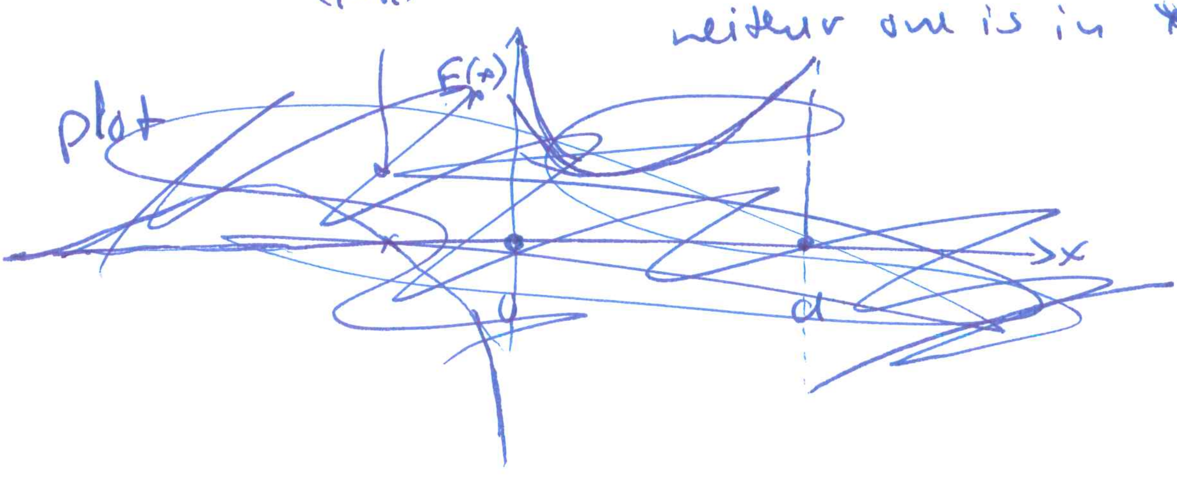
$$x^2 + 2xd - d^2 = 0$$

$$x = \frac{-2d \pm \sqrt{4d^2 + 4d^2}}{2} = -d \pm \sqrt{2}d$$

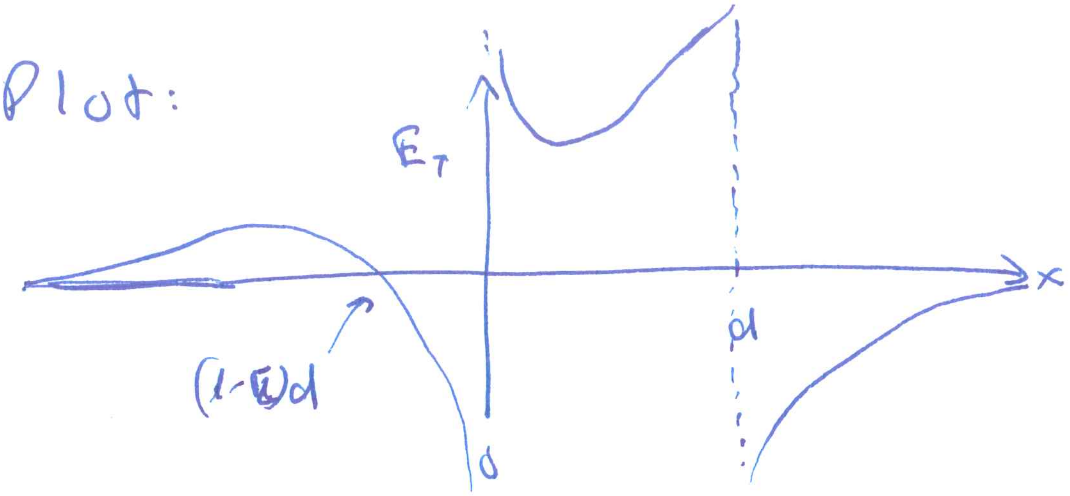
neither one is in $x > d$

$(1-\sqrt{2})d$

plot



Plot:



10.)

E-field is zero at one pt. only

~~E-field~~ E-field in between:

$$\frac{5Q}{4\pi\epsilon_0 x^2} - \frac{3Q}{4\pi\epsilon_0 (x-d)^2} = 0$$

$$\Downarrow$$

$$\frac{5}{x^2} = \frac{3}{(x-d)^2}$$

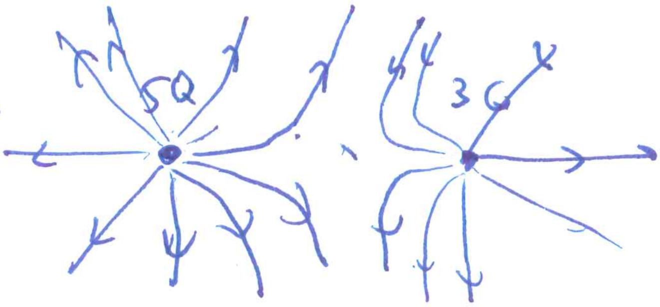
$$5x^2 - 10xd + 5d^2 = 3x^2$$

$$2x^2 - 10xd + 5d^2 = 0$$

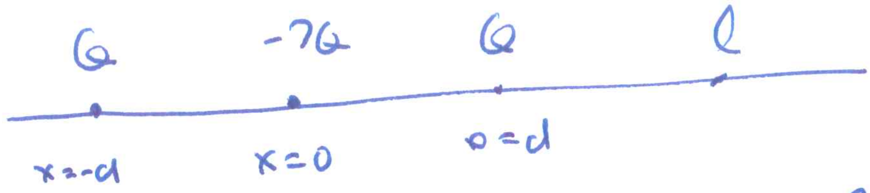
$$x = \frac{10d \pm \sqrt{100d^2 - 40d^2}}{4}$$

$$= \left(\frac{10 \pm \sqrt{60}}{4}\right) d$$

field lines



11.)



B-field: $E = \frac{Q}{4\pi\epsilon_0 (l+d)^2} - \frac{2Q}{4\pi\epsilon_0 l^2} + \frac{Q}{4\pi\epsilon_0 (l-d)^2}$

limit $l \to \infty$

$$E = \frac{Q}{4\pi\epsilon_0 l^2 (1+\frac{d}{l})^2} - \frac{2Q}{4\pi\epsilon_0 l^2} + \frac{Q}{4\pi\epsilon_0 l^2 (1-\frac{d}{l})^2}$$

$d/l \to 0$

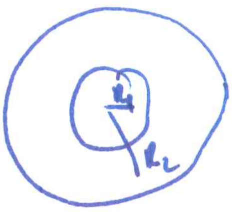
expand: $f(x) = \frac{1}{1+x} \Rightarrow f'(x) = -\frac{1}{(1+x)^2}$ | $f''(x) = \frac{2}{(1+x)^3}$

$$f(x) \sim 1 - x + x^2$$

$$B = \frac{Q}{4\pi\epsilon_0 l^2}\left(1 - \cancel{\frac{d}{l}}\right) - \frac{2Q}{4\pi\epsilon_0 l^2} + \frac{Q}{4\pi\epsilon_0 l^2}\left(1 + \cancel{\frac{d}{l}}\right)$$

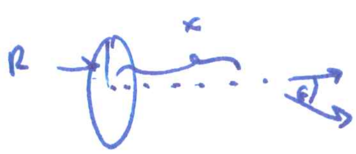
$$+ \frac{2Q}{4\pi\epsilon_0 l^2}\frac{d^2}{l^2} = \frac{2Qd^2}{4\pi\epsilon_0 l^4}$$

12.)



surface charge density σ

one ring of thickness dR at radius R



$$dE = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma\, dR\, dl}{(R^2+x^2)} \frac{x}{(R^2+x^2)^{1/2}} dR$$

$$= \frac{\sigma R x 2\pi}{4\pi\epsilon_0 (R^2+x^2)^{3/2}}$$

$$dE = \frac{\sigma R}{2\epsilon_0} \frac{x}{(x^2+R^2)^{3/2}} dR$$

$$E = \int_{R_1}^{R_2} \left(\frac{\sigma x}{4\epsilon_0}\right) \frac{2R\,dR}{(x^2+R^2)^{3/2}}$$

$$u = R^2 + x^2$$

$$du = 2R\,dR$$

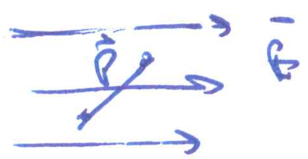
$$E = \int_{R_1^2+x^2}^{R_1^2+x^2} \left(\frac{\sigma x}{4\epsilon_0}\right) \frac{du}{u^{3/2}}$$

$$= \left(\frac{\sigma x}{4\epsilon_0}\right) \left[-\frac{1}{u^{1/2}}\right]_{R_1^2+x^2}^{R_2^2+x^2}$$

$$= \frac{\sigma x}{4\epsilon_0} \left[\frac{1}{(R_1^2+x^2)^{1/2}} - \frac{1}{(R_2^2+x^2)^{1/2}}\right]$$

13.) p, I, E small oscillations

$$\vec{\tau} = \vec{p} \times \vec{E} = -pE\sin\theta$$



$$I\ddot{\theta} = -pE\sin\theta$$

small oscillations: $\sin\theta \sim \theta$

$$I\ddot{\theta} = pE\theta$$

$$\Rightarrow \theta(t) = A e^{i\omega t}$$

$$\ddot{\theta}(t) -$$

$$\omega^2 I = pE$$

$$\boxed{\omega = \sqrt{\frac{pE}{I}}}$$