Solutions: Problem Set 10

1.) (a) ~~$V = IR$~~ $\quad v = V\cos\omega t$

$i = I\cos\omega t$

$v = iR \Rightarrow V = IR$

phase: 0

(b) $v = L\frac{di}{dt}$ $\quad \frac{di}{dt} = I\sin\omega t(-\omega)$ (using $i = I\cos\omega t$)

$= -\omega I\sin\omega t$

$= \omega I\cos(\omega t + \frac{\pi}{2})$

since $\cos(\omega t + \frac{\pi}{2})$

$= \cos\omega t\cos\frac{\pi}{2} - \sin\omega t\sin\pi/2$

$= -\sin\omega t$

$\to 0$

$v = I\omega L\cos(\omega t + \frac{\pi}{2})$

phase: $\pi/2$

(c) capacitance:

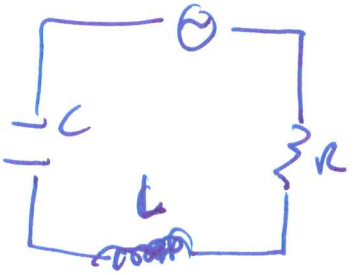
$v = \frac{q}{C}$ $\quad q = \int_0^t I\cos\omega t'dt' + q(0)$

$= \frac{I\sin\omega t'}{\omega}\Big|_0^t = \frac{I\sin\omega t}{\omega}$

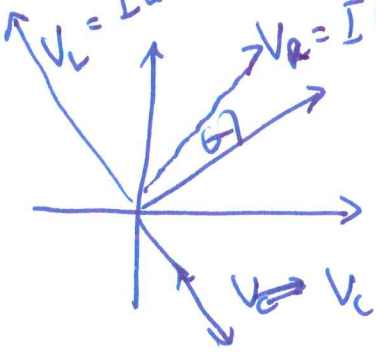
$v = \frac{I}{\omega C}\sin\omega t = \frac{I}{\omega C}\cos(\omega t - \frac{\pi}{2})$

phase: $-\pi/2$

2.)



$\Rightarrow$



~~$V = V_R + V_L + V_C$~~

vector addition: $\frac{V}{\sqrt{2}} = \frac{I}{\sqrt{2}}\underbrace{\sqrt{(X_L - X_C)^2 + R^2}}_{Z}$

$V_{rms} = I_{rms}R$

phase angle: $\tan\theta = \frac{X_L - X_C}{R}$ (2)

3.) $I = 5A \quad V = 160\,V$

impedance: $V = IZ \quad Z = \frac{160}{5} = 32\,\Omega \qquad \cos\varphi = \frac{R}{Z}$

dissipated power: $P = \frac{I^2 R}{2}\cos\varphi = 600\,W$

~~$P = iv = I_0 \cos\omega t$~~

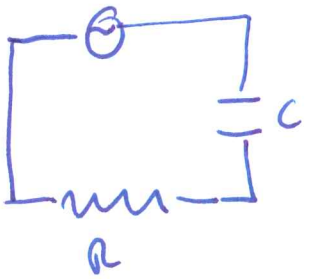
$= \frac{I^2 Z^2}{2R} = 600$

~~$6 \cdot 4 \times$~~ $48 \cdot 4.4$

$R^2 = \frac{600 \times 64}{25} = 24 \times 64$

$R = 4\sqrt{48} = 16\sqrt{3}$

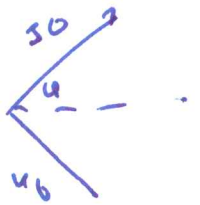
4.)



$220V = V_{rms} = \frac{V}{\sqrt{2}}$

$X_C = 40\Omega$

$R = 30\,\Omega$



(2) impedance: $Z = \sqrt{40^2 + 30^2} = 50\,\Omega$

4) $I_{rms} = \frac{220}{50} = 4.4\,A$

$27 \cdot 22$
44
66

6.) $\varphi = -\arctan\frac{4}{3}$

(.) power loss: $\frac{I^2}{2} R \frac{3}{5} = \left(\frac{22}{5}\right)^2 \frac{30 \cdot 3}{10}$

$\frac{9 \cdot (22)^4}{25} = \frac{9 \times 462}{25}\,W =$

$166\,W$

$462 : 25 = 18.4 \cdot 9$
212
120

$18.4 \cdot 9$
165.6

5.) $\nu = 60\,Hz$ $60/sec$

$\omega = 60 \cdot 2\pi$ rad/sec

$\omega = 2\pi \cdot 60$

$\omega = \frac{1}{\sqrt{LC}} = 60 \cdot 2\pi$

$Z = \sqrt{(\omega L - \frac{1}{\omega C})^2 + R^2}$ ω resonance

$Z = 8\,\Omega = R$

$Z = \sqrt{(\omega L - \frac{1}{\omega C})^2 + 64}\ \Omega$

$\frac{3600 \cdot 4}{14400}$

$\nu = 80\,Hz \Rightarrow \omega = 80 \cdot 2\pi$

$10 = \sqrt{(2\pi \cdot 80 L - \frac{1}{(2\pi \cdot 80) C})^2 + 64}$

$6 = (2\pi \cdot 80) L - \frac{1}{(2\pi \cdot 80) C}$

$\Rightarrow 2\pi \cdot 60 = \frac{1}{\sqrt{LC}}$

$4\pi^2 \cdot 3600 = \frac{1}{LC}$

$14400\pi^2 = \frac{1}{LC}$

$L = \frac{1}{14400\pi^2 C}$

$= \frac{2\pi \cdot 80}{14400\pi^2 C} - \frac{1}{(2\pi \cdot 80) C}$

$14400 \div 16 = 9$

$= \frac{1}{90\pi C} - \frac{1}{160\pi C}$

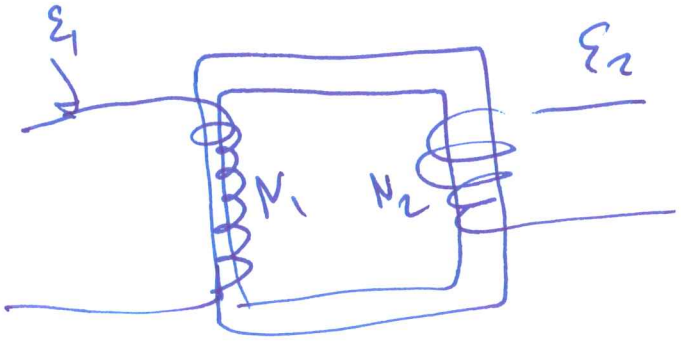
$6 = \frac{1}{9\pi C} - \frac{1}{16\pi C} = (\frac{1}{9} - \frac{1}{16}) \frac{1}{\pi C}$

$C = (\frac{1}{9} - \frac{1}{16}) \frac{1}{60\pi} = \frac{16 - 9}{144} \frac{1}{60\pi}$

$C = \frac{7}{144 \cdot 60\pi} \approx 2.6 \times 10^{-4}\ F$

$L = \frac{144 \cdot 60\pi}{14400\pi^2} = \frac{6}{10\pi} = .19\ H$

6.)



$$\mathcal{E}_1 = N_1 \frac{d\Phi_B}{dt}$$

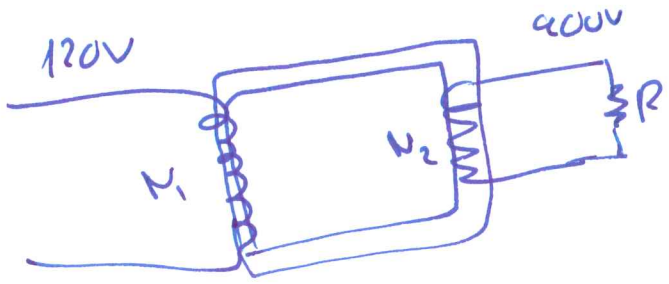
$$\mathcal{E}_2 = N_2 \frac{d\Phi_B}{dt}$$

$$\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2} \qquad 1800 \div 12 = 15$$

$$\frac{120}{1800} = \frac{1800}{N_2} \Rightarrow N_2 = \frac{1800 \cdot 10}{12}$$

$$= 150 \cdot 10 = 1500$$

7.)



$$R = \frac{400}{5} = 300\ \Omega$$

$$\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2} \Rightarrow \qquad \frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \Rightarrow \frac{400}{120} = \frac{N_2}{N_1} = 7.5$$

power dissipated: $P = I_1^2 R$

$$= \frac{V^2}{R} = \frac{\mathcal{E}_1^2}{R}$$

$$P = \frac{\mathcal{E}_1^2 N_2^2}{N_1^2 R} = \frac{\mathcal{E}_1^2}{R_1}$$

$$R_1 = R \frac{N_1^2}{N_2^2} = 300 \cdot 7.5 \cdot 7.5\ \Omega$$

$$\begin{array}{r} 300 \cdot 7.5 \\ 2100 \\ 1500 \\ \hline 2250.0 \cdot 7.5 \\ \hline 16750 \\ 11250 \\ \hline 16875.0 \end{array}$$

$$I = \frac{120}{16875}\ A$$