Topological aspects of ideal conduction

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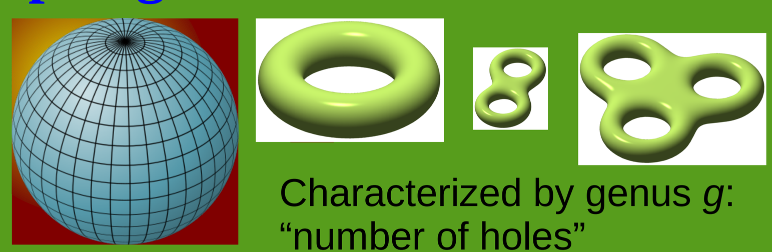
Topological invariants



Characterized by genus g: "number of holes"

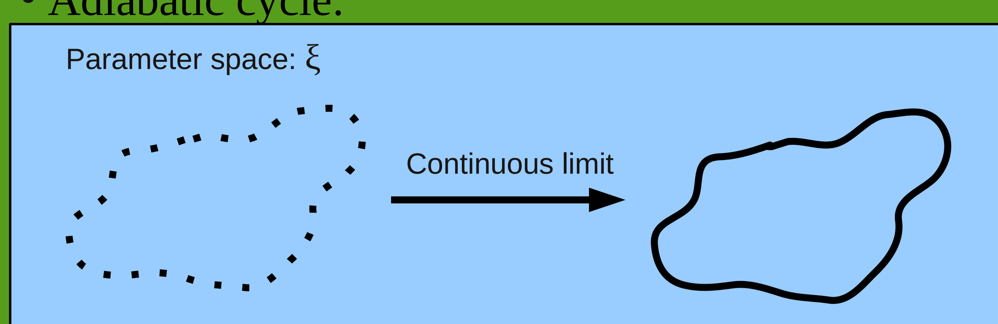
- Genus g can also be cast in terms of the curvature K over the surface:

$$n(1-g) = (1/2\pi)\int_S K dS$$

- Integrals over curvature: **topological invariants**

Berry phase & Berry curvature

- Adiabatic cycle:



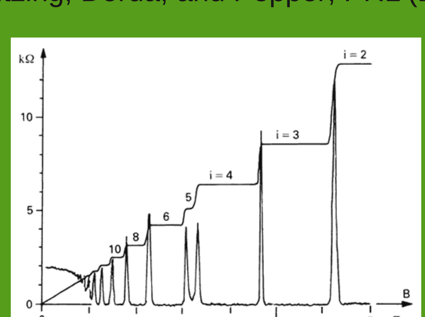
- Discrete Berry phase: $\phi_B = \mathrm{Im}\ln\prod_i \langle\Psi(\xi_i)|\Psi(\xi_{i+1})\rangle$
- Continuous Berry phase: $\phi_B = \oint d\xi \cdot \langle\Psi(\xi)|\nabla_\xi|\Psi(\xi)\rangle$
- Stokes theorem → Berry phase becomes a curvature integral: $\phi_B = \int d\sigma_\xi \cdot \nabla_\xi \times \langle\Psi(\xi)|\nabla_\xi|\Psi(\xi)\rangle$

Topological systems

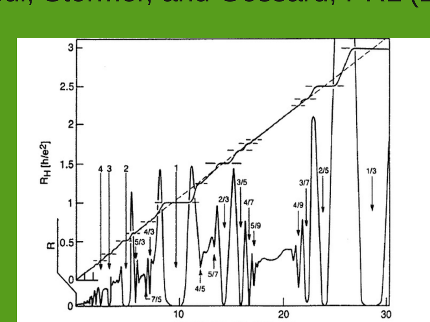
- **Characterized by topological invariant**
- **Quantization of transport related quantities**
- **Edge currents at interfaces** (Halperin, PRB 1984)
- Examples: Quantum Hall systems, topological insulators

Quantum Hall effect & TKNN invariant

(von Klitzing, Dorda, and Pepper; PRL (1980))



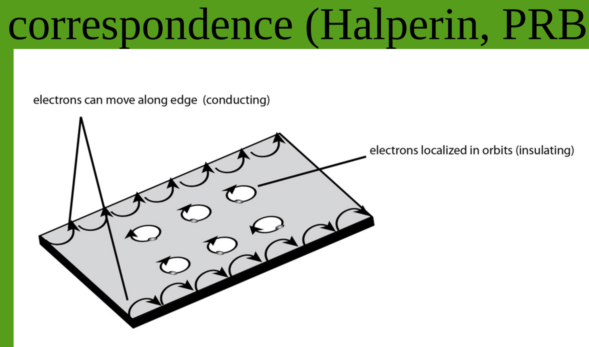
(Tsui, Stormer, and Gossard; PRL (1982))



- Quantized Hall conductance: $\sigma_{xy} = p\dfrac{e^2}{h}$ $\quad \sigma_{xy} = \dfrac{p}{q}\dfrac{e^2}{h}$
- Thouless, Kohmoto, Nightingale, and den Nijs (PRL, 1982): σ_{xy} **is a topological invariant → quantization**

$$\sigma_{xy} = \frac{ie^2}{2\pi h}\sum_{m=1}^{n_{occ}}\int\int dk_x dk_y \left(\langle\partial_{k_x}u_m|\partial_{k_y}u_m\rangle - \langle\partial_{k_y}u_m|\partial_{k_x}u_m\rangle\right)$$

- Bulk-boundary correspondence (Halperin, PRB, 1982): **edge currents**

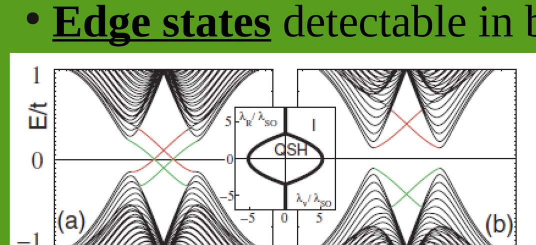


Kane-Mele model: edge states

- Hamiltonian (Kane and Mele, PRL 2005):

$$H = -t\sum_{\langle i,j\rangle} c_i^\dagger c_j + i\lambda_{SO}\sum_{\langle\langle i,j\rangle\rangle}\nu_{ij}c_i^\dagger\sigma_z c_j + i\lambda_R\sum_{\langle i,j\rangle}c_i^\dagger(\mathbf{s}\times\hat{\mathbf{d}}_{ij})_z c_j + \lambda_\nu\sum_i \xi_i c_i^\dagger c_i$$

- **Edge states** detectable in band structure:



Topological invariant:

$$I = \int d\mathbf{k}\cdot\nabla_{\mathbf{k}}\ln(P(\mathbf{k})+i\delta)$$

Drude weight/Ideal conduction

- Derived in a seminal paper by Kohn (PR, 1964) as the criterion to distinguish conductors from insulators
  - Related to many-body localization
- Measure of ideal (non-diffusive) conduction:

$$\sigma(\omega) = D\delta(0) + \sigma_{\mathrm{reg}}(\omega)$$

Drude weight as equilibrium susceptibility

- Hamiltonian: $H(\Phi) = \sum_{i=1}^{N}\dfrac{(\hat{p}_i+\Phi)^2}{2m} + \hat{V}$

Φ represents perturbation (electric field)

- **Current:** $J(\Phi) = \dfrac{\partial E(\Phi)}{\partial\Phi} = \langle\Psi(\Phi)|\left[\sum_i\dfrac{(\hat{p}_i+\Phi)}{m}\right]|\Psi(\Phi)\rangle$
- **Drude weight:** $D = \dfrac{1}{V}\left[\dfrac{\partial^2 E_0(\Phi)}{\partial\Phi^2}\right]_{\Phi=0}$

Relation to other ideal transport quantities

Drude weight:	Meissner weight:	Superfluid weight:
$D = \frac{1}{V}\left[\frac{\partial^2 E_0(\Phi)}{\partial\Phi^2}\right]_{\Phi=0}$	$n^{(s)} = \frac{1}{V}\left[\frac{\partial^2 E(\Phi)}{\partial\Phi^2}\right]_{\Phi=0}$	$I^{(s)} = \left[\frac{\partial^2 E^{(s)}(\Phi)}{\partial\Phi^2}\right]_{\Phi=0}$

Drude weight as a topological invariant

- Similar to the modern theory of polarization (King-Smith & Vanderbilt PRB 1993; Resta RMP 1994) the current can be expressed as a geometric phase (Thouless, PRB 1983):

$$J(\Phi) = \frac{N\Phi}{m} - \frac{i}{ma}\int_0^a dX\langle\Psi(X)|\frac{\partial}{\partial X}|\Psi(X)\rangle$$

- Taking the derivative with respect to Φ, and averaging over the Brillouin zone results in:

$$D = \frac{N}{m} - \frac{i}{2\pi m}\int_{BZ}\int_0^a dK dX\left(\left\langle\frac{\partial}{\partial K}\Psi\middle|\frac{\partial}{\partial X}\Psi\right\rangle - \left\langle\frac{\partial}{\partial X}\Psi\middle|\frac{\partial}{\partial K}\Psi\right\rangle\right)$$

- The second term of D is the integral over a curvature: hence D is a **topological invariant**. The integral is over the variables K,X the total momentum and the total position ("phase space").
- For a system with a magnetic field, the Hall conductance is easily recovered (in this case X becomes the transverse momentum K_y)
- The topological invariant derived by Thouless in the context of adiabatic charge pumping can also be derived (Thouless, PRB 1983):

$$C = \frac{i}{2\pi}\int_0^T dt\int_{BZ}d\Phi\left(\left\langle\frac{\partial}{\partial\Phi}\Psi\middle|\frac{\partial}{\partial t}\Psi\right\rangle - \left\langle\frac{\partial}{\partial t}\Psi\middle|\frac{\partial}{\partial\Phi}\Psi\right\rangle\right)$$

Quantization

- Flux quantization inside a cavity of a superconductor (Deaver and Fairbank, PRL 1961; Byers and Yang, PRL 1961)
- From London equation: Meissner weight should be fractionally quantized

$$\frac{ie\hbar}{m}\oint\langle\Psi|\nabla_{\mathbf{r}}|\Psi\rangle\cdot d\mathbf{l} = -\frac{n_s e^2}{mc}\Phi_B$$

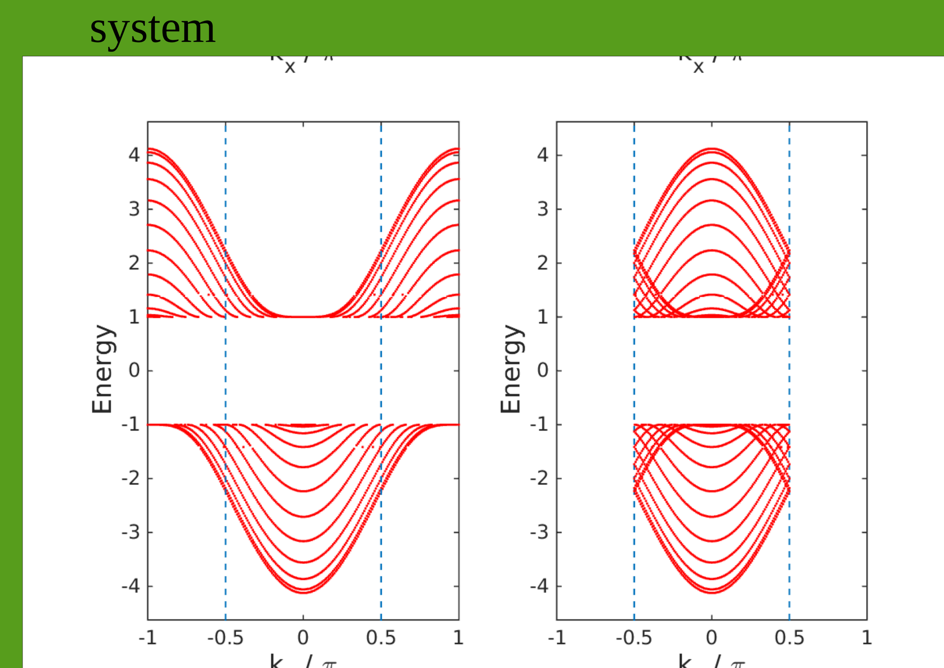
- M. Oshikawa (PRL 2000, PRL 2003) has shown that the Drude weight can only be zero for fillings of rational fractions

Edge states at conductor/CDW interface

- To investigate edge state we consider an interface between a simple spinless tight-binding model and a charge density wave
- Tight-binding Hamiltonian: $\hat{H}_{TB} = -t\sum_{\langle i,j\rangle}(c_i^\dagger c_j + c_j^\dagger c_i)$
- CDW Hamiltonian: $\hat{H}_{CI} = \hat{H}_{TB} + U\sum_i \xi_i \hat{n}_i$

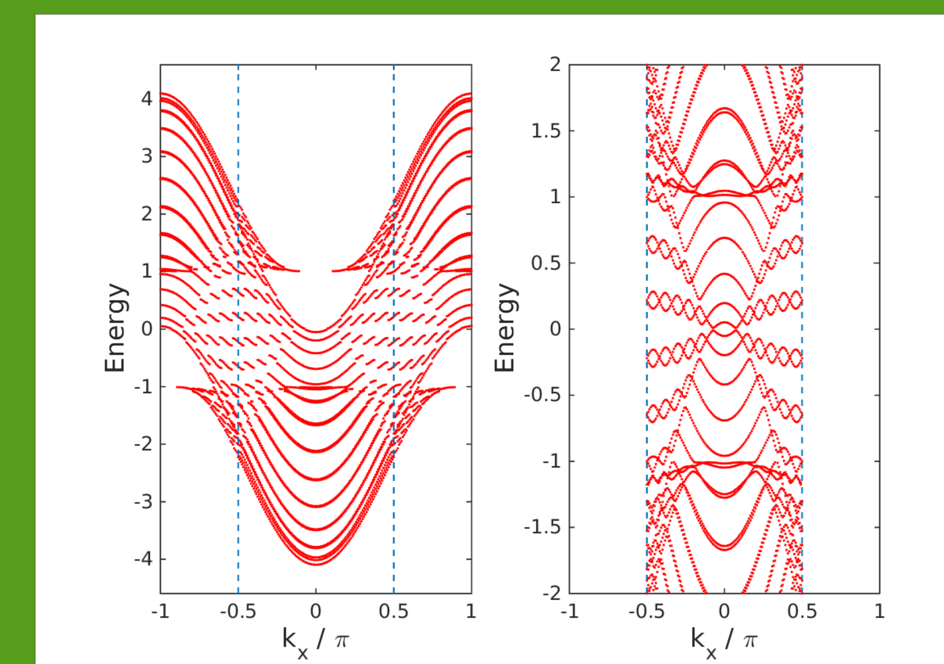
$\xi_i = \pm 1$ arranged in a checkerboard fashion

- Band structure of CDW projected along one axis: gapped system

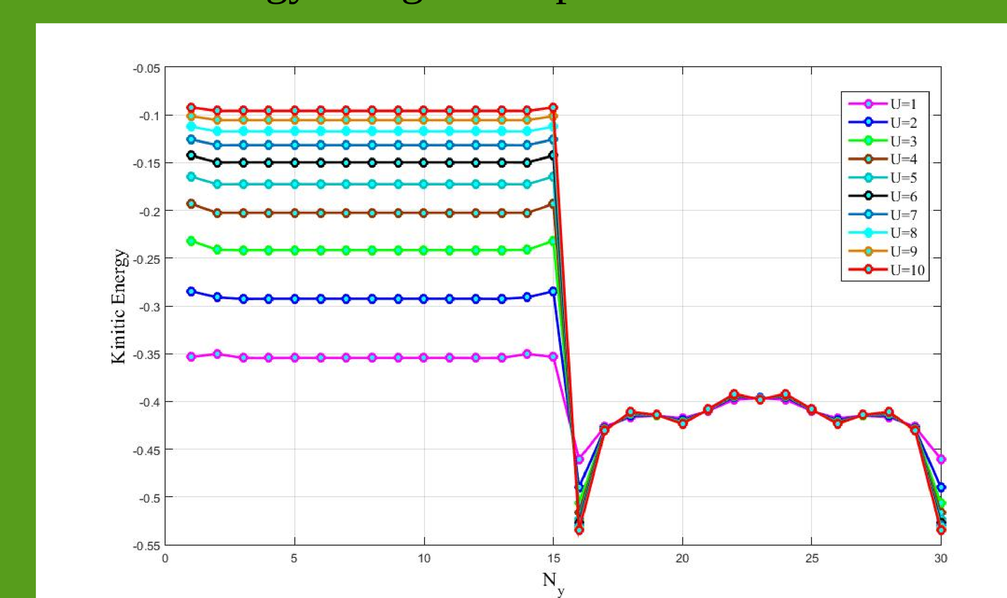


Edge states (cont'd)

- Band structure of system with TB/CDW interface



- Kinetic energy along bonds parallel to the interface



- Current/resolved Drude weight show similar behavior

Transport coefficients and ODLRO

- How can one distinguish transport coefficients D, $n^{(s)}$, and $I^{(s)}$?
- Definition of p-current based on generator of p-particle translations:

$$J_p(\Phi) = \frac{N}{m}\Phi + \lim_{\Delta X\to 0}\frac{(N/p)}{m\Delta X}\mathrm{Im}\ln\mathrm{Tr}\left\{\hat{\rho}_p\exp\left(i\Delta X\sum_{j=1}^{p}\hat{p}_j\right)\right\}$$

Note: ΔX → 0 limit leads to the same expression, however the quantities J_p are distinct: they depend on reduced density matrices (RDM) of different orders

- p-particle transport coefficient

$$D_p = \left[\frac{\partial J_p(\Phi)}{\partial\Phi}\right]_{\Phi=0}$$

eigenstates of the p-particle momentum make finite contributions to D_p

- RDM and ODLRO:

$$\lim_{|x_1-x_1'|\to\infty}\rho_1(x_1;x_1') = \text{finite}$$

- Yang (RMP, 1962): if ODLRO occurs in the p-particle RDM, than it will also occur in all larger order RDMs
- Examples: $p=1$ → BEC in bosonic superfluids; $p=2$ → BEC in Cooper paired systems;

- Transport coefficients D_p behave in a similar manner to ODLRO

If for a particular m D_m is finite, all D_p with $p>m$ are also finite.

- Minimum value of p for which D_p is finite determines the flux quantization rule associated with conduction:

$$\Phi_B = \frac{hc}{pe}q$$

- Classes of ideal conductors:
  - If $p<<N$ system exhibits ideal conduction with flux quantization
  - If $p\sim N$ system exhibits ideal conduction with no flux quantization
  - If all D_p are zero: system is an insulator or Ohmic conductor

- **Surprising result: D_N for the Hubbard model at half-filling is finite → Fröhlich superconductor**

- **Acknowledgment:** We acknowledge financial support from the Turkish agency for basic research TÜBITAK grant no. 113F334 and 112T176.

References:

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