Name Surname: November 8, 2010

MATH 101-007 Quiz 6

CHOOSE ONLY ONE OF THE FOLLOWING QUESTIONS

Question 1. Evaluate $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} = L$

Question 2. Find the largest constant k such that $x + \frac{3}{x} \geq k$ for all $x > 0$.

Question 1. We have the form $\frac{0}{0}$. So we use l'Hôpital's rule.

$$L = \lim_{x\to 0} \frac{-\sin(\sin x)\cos x + \sin x}{4x^3} \quad \frac{0}{0} \text{ l'Hôpital's rule}$$

$$= \lim_{x\to 0} \frac{-\cos(\sin x)\cos^2 x + \sin(\sin x)\sin x + \cos x}{12x^2}$$

$$= \lim_{x\to 0} \frac{-\cos(\sin x)\cos^2 x + \cos x}{12x^2} + \lim_{x\to 0} \frac{\sin(\sin x)\sin x}{12x^2}$$

$$= \lim_{x\to 0} \frac{-\cos(\sin x)\cos x + 1}{12x^2} \cdot \underbrace{\left(\lim_{x\to 0} \cos x\right)}_{=1} + \frac{1}{12}\underbrace{\left(\lim_{x\to 0} \frac{\sin(\sin x)}{x} \cdot \frac{\sin x}{x}\right)}_{=1}$$

$$= \frac{1}{12} + \underbrace{\lim_{x\to 0} \frac{-\cos(\sin x)\cos x + 1}{12x^2}}_{\text{call this } K}$$

call this K. It's in the form $\frac{0}{0}$. Use l'Hôpital's rule for K.

$$K = \lim_{x\to 0} \frac{\sin(\sin x)\cos^2 x + \cos(\sin x)\sin x}{24x}$$

$$= \frac{1}{24}\left(\lim_{x\to 0} \frac{\sin(\sin x)}{x}\cos^2 x + \lim_{x\to 0} \cos(\sin x)\frac{\sin x}{x}\right)$$

$$= \frac{1}{24}(1+1) = \frac{1}{12}.$$

So $L = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$.

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CHOOSE ONLY <u>ONE</u> OF THE FOLLOWING QUESTIONS

Question 1. Evaluate $\lim_{x\to 0} \dfrac{\cos(\sin x) - \cos x}{x^4}$.

Question 2. Find the largest constant k such that $x + \dfrac{3}{x} \geq k$ for all $x > 0$.

<u>Question 2.</u> Let $f(x) = x + \frac{3}{x}$, $x>0$.

$f'(x) = 1 - \frac{3}{x^2} = 0 \Rightarrow x = \sqrt{3}$, $\cancel{x = -\sqrt{3}}$ not in the domain.

$x = \sqrt{3} \Rightarrow f(x) = \sqrt{3} + \frac{3}{\sqrt{3}} = 2\sqrt{3}$.

$\lim_{x\to 0} f(x) = \lim_{x\to 0} \left(x + \frac{3}{x}\right) = \infty$

$\lim_{x\to \infty} f(x) = \lim_{x\to \infty} \left(x + \frac{3}{x}\right) = \infty$.

Thus $f(x)$ (for $x>0$) has absolute minimum at $x = \sqrt{3}$ and absolute minimum value is $2\sqrt{3}$. Then $f(x) \geq 2\sqrt{3}$ for all $x>0$, that is $x + \frac{3}{x} \geq 2\sqrt{3}$ for all $x>0$.

Since $\sqrt{3} + \frac{3}{\sqrt{3}} = 2\sqrt{3}$, $2\sqrt{3}$ is the largest such constant k.