Name Surname: November 1, 2010

MATH 101-007 Quiz 5

Question. By considering the sign of y', y'', all asymptotes, and making a complete table, sketch the graph of the following curve:

$$y = \frac{x^2 + 3x + 18}{x + 2}.$$

Note: $y' = \dfrac{x^2 + 4x - 12}{(x+2)^2}$ and $y'' = \dfrac{32}{(x+2)^3}$. You may use them without calculating them.

Solution. Domain : all $x \neq -2$.

$\lim_{x \to -2^+} \dfrac{x^2+3x+18}{x+2} = \infty$, $\lim_{x \to -2^-} \dfrac{x^2+3x+18}{x+2} = -\infty$.

So $x=-2$ is vertical asymptote.

$\dfrac{x^2+3x+18}{x+2} = x+1+\underbrace{\dfrac{16}{x+2}}_{\downarrow 0 \text{ as } x \to \mp\infty}$. So $y = x+1$ is oblique asymptote as $x \to \mp\infty$.

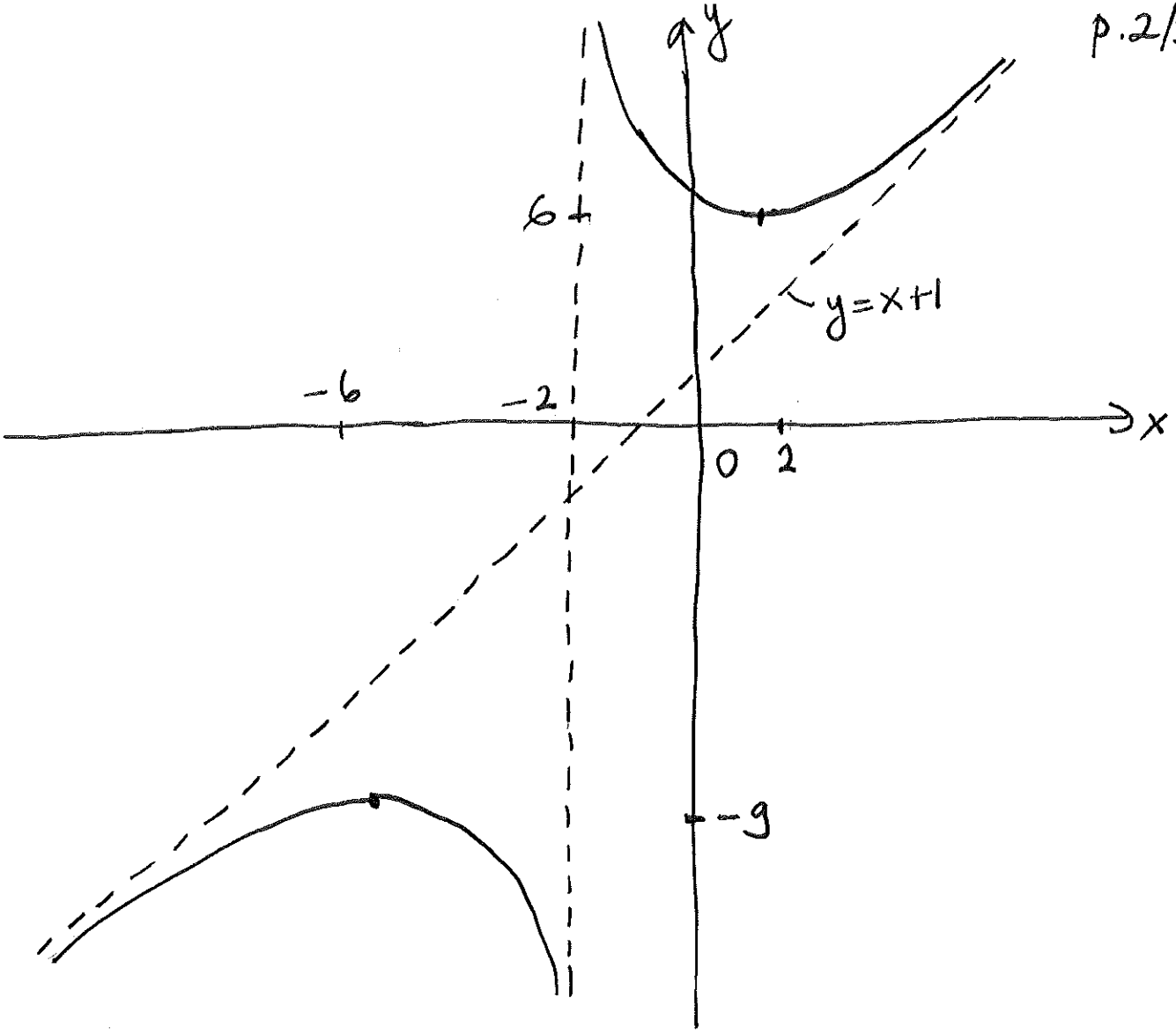
$y' = \dfrac{x^2+4x-12}{(x+2)^2} = \dfrac{(x+6)(x-2)}{(x+2)^2}$

x		-6		-2		2
y'	$+$	0	$-$	‖	$-$	0 $+$

$y'' = \dfrac{32}{(x+2)^3}$

x		-2	
y''	$-$	‖	$+$

x	$-\infty$		-6		-2		2		∞
y'		$+$	0	$-$	‖	$-$	0	$+$	
y''	$-$			$-$	‖	$+$		$+$	
y	$x+1$	↗	-9	↘	$-\infty$ ‖ ∞	↘	7	↗	$x+1$



$$y = \frac{x^2+3x+18}{x+2}$$