Name Surname: October 4, 2010

MATH 101-007 Quiz 2

Question 1. Show that there are at least two real numbers c such that

$$c^4 + c^3 + c^2 + 10 \sin^2 c = 119.$$

Explain and support your answer by well-known theorem(s).

Solution. Let $f(x) = x^4 + x^3 + x^2 + 10\sin^2 x$.

Then f is continuous at every point.

(1) $f(0) = 0 < 119$.

$f(4) = 256 + 64 + 16 + \underbrace{10\sin^2 4}_{\geq 0} > 256 > 119$.

Since f is continuous on $[0,4]$ and $f(0) < 119$, $f(4) > 119$; by the Intermediate Value Theorem, there is at least one point c between $x=0$ and $x=4$ such that $f(c) = 119$.

(2) $f(0) = 0 < 119$

$f(-4) = \underbrace{256 - 64 + 16}_{208} + 10\underbrace{\sin^2(-4)}_{\geq 0} \geq 208 > 119$

Since f is continuous on $[-4,0]$; by the Intermediate Value Theorem, there is at least one point c between $x=-4$ and $x=0$ such that $f(c) = 119$.