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MATH 101-007 Quiz 1

Question 1. A certain function f has the property that $\lim_{x\to 0}\frac{f(x)}{x}=2$. Find $\lim_{x\to 0}\frac{f(3x)}{x}$. Explain.

Solution. Let $3x=t$. Then $t\to 0$ as $x\to 0$.

$$\lim_{x\to 0}\frac{f(3x)}{x}=\lim_{t\to 0}\frac{f(t)}{t/3}=3\lim_{t\to 0}\frac{f(t)}{t}$$

$$=3\cdot 2=6.$$

Question 2. Evaluate

$$\lim_{x\to 1}\frac{|x^2+2x-5|-|3x^2+6x-7|}{|2x^2-5x+9|-|3x-9|}.$$

As $x\to 1$, we have $x^2+2x-5\to -2$, so $x^2+2x-5<0$,
$3x^2+6x-7\to 2$, so $3x^2+6x-7>0$,
$2x^2-5x+9\to 6$, so $2x^2-5x+9>0$,
$3x-9\to -6$, so $3x-9<0$.

Then $$\lim_{x\to 1}\frac{|x^2+2x-5|-|3x^2+6x-7|}{|2x^2-5x+9|-|3x-9|}=$$

$$\lim_{x\to 1}\frac{-(x^2+2x-5)-(3x^2+6x-7)}{(2x^2-5x+9)-(-(3x-9))}=\lim_{x\to 1}\frac{-4x^2-8x+12}{2x^2-2x}$$

$$=\lim_{x\to 1}\frac{-4\cancel{(x-1)}(x+3)}{2\cancel{(x-1)}x}=\lim_{x\to 1}\frac{-4(x+3)}{2x}=-8.$$