February 28, 2014

MATH 215 Homework 4

Turn in by March 7, 2014 until 10:30 am.

Question 1. (20×2=40 points) The following sets are considered in the metric space (X, d), where $X = \mathbb{R}^2$ and d is the Euclidean metric. For each of these sets, write the following: **(no explanation is required)**

(i) Write $\text{int}E$.
(ii) Write E'.
(iii) Write $\overline{E}$.
(iv) Write whether E is open, closed or neither.

Sets.
a. $E = \{(x, y) : 0 < x, \text{ and } y \geq \frac{1}{x}\}$.
b. $E = \{(x, y) : -1 < x < 1\}$.
c. $E = \{(x, y) : x \in \mathbb{Q}\}$.

d. $E = \left\{\left(\frac{1}{n}, \frac{1}{n^2}\right) : n \in \mathbb{N}\right\} \cup \{(0, 0)\}$.

e. $E = \left\{\left(\frac{1}{n}, \frac{1}{m}\right) : n, m \in \mathbb{N}\right\}$.

Question 2. (20+20=40 points) Let A and B be two arbitrary subsets of a metric space (X, d). Prove the following:
(a) $\text{int}(A \cap B) = \text{int}A \cap \text{int}B$.
(b) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

3. (20 points) Given a set E in a metric space (X, d), we define the **exterior** $\text{ext}E$ of E as $\text{ext}E = \text{int}(\mathcal{C}E)$. Prove that

$$\text{ext}E = \mathcal{C}\overline{E}.$$