

Date: 12 July 2010 Time: 08:40-10:30

NAME SURNAME:.....

STUDENT NO:.....

MATH 114 MIDTERM 2

IMPORTANT

1. This exam consists of 6 questions of different weights.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.
5. Mobile telephones must be shut off.

Please do not write anything below this line.

1	2	3	4	5	6	TOTAL
17	17	17	17	15	17	100

Q 1. (17 points) Let f be a function of two variables with continuous first and second order partial derivatives, and let g be a differentiable function of one variable. Let $z = f(x^2g(y), x + y^2)$. Given that

$$g(3) = -1, \quad g'(3) = -3, \quad f_1(-4, 11) = 2, \quad f_{11}(-4, 11) = -4, \quad f_{12}(-4, 11) = -2, \quad f_{22}(-4, 11) = 3,$$

find

$$\left. \frac{\partial^2 z}{\partial y \partial x} \right|_{(x,y)=(2,3)}.$$

Q 2. (17 points) Find the values of the constants a, b and c such that the directional derivative of $f(x, y, z) = axy^2 + byz + cz^2x^3$ at the point $(1, 2, -1)$ has a maximum value of 64 in a direction parallel to the z -axis.

Q 3. (17 points) Find and classify all the critical points of the following function:

$$f(x, y) = x^2y + y^3 - 3y^2.$$

Q 4. (17 points) Find the maximum and minimum values of $f(x, y, z) = xy + z^2$ subject to the constraint $\frac{x^2}{16} + \frac{y^2}{81} + \frac{z^2}{4} = 1$.

Q 5. (15 points) Evaluate the following iterated integral:

$$\int_0^1 dx \int_{x^{1/3}}^1 \frac{1}{\sqrt{1+y^4}} dy.$$

Q 6. (17 points) Evaluate the following iterated integral:

$$\int_0^2 \int_0^{\sqrt{1-(y-1)^2}} \sqrt{4-x^2-y^2} \, dx \, dy.$$