Date: 28 June 2010 Time: 08:40-10:30

NAME SURNAME:..

STUDENT NO:..

MATH 114 MIDTERM 1

IMPORTANT

1. This exam consists of 5 questions of equal weight.

2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.

3. Show all your work. Correct answers without sufficient explanation might not get full credit.

4. Calculators are not allowed.

5. Mobile telephones must be shut off.

Please do not write anything below this line.

1	2	3	4	5	TOTAL
20	20	20	20	20	100

Q 1. Let R be the region in the first quadrant bounded by $y = \sqrt{x}$, the x-axis, and the line $x = 4$. Assume R is rotated about the line $y = -1$ and a solid is generated.

(a) Draw the region R.

(b) Set up the integral for the volume of the solid by using the slicing method. Do not evaluate the integral.

(c) Set up the integral for the volume of the solid by using the cylindrical shells method. Do not evaluate the integral.

Q 2. Consider the following table.
In column I you're given some series $\sum a_n$.
In column II, write the name of the test that you would use to check whether the series $\sum a_n$ is convergent.
If the test you named in column II, is Comparison Test or Limit Comparison Test, write the comparison series $\sum b_n$ in column III. Otherwise leave this column empty.
In column IV, write your conclusion (as **convergent** or **divergent**) about the given series $\sum a_n$ in column I.
No other explanation is required.

Column I	Column II	Column III	Column IV
Series $\sum a_n$	Which test?	If the test in Column II is Comparison or Limit Comparison, write the series $\sum b_n$	Is the series $\sum a_n$ convergent or divergent?
$\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}}{2^n}$			
$\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n+5}{9n-7}$			
$\displaystyle\sum_{n=1}^{\infty} \frac{3+\sqrt{n}}{\sqrt{n^2+5n+2}}$			
$\displaystyle\sum_{n=3}^{\infty} \frac{1}{n\,\ln n\,(\ln(\ln n))^3}$			

Q 3. Let

$$F(x) = \int_0^x \sin(t^2)\, dt.$$

Find a polynomial $P(x)$ that approximates $F(x)$ with $|\text{error}| < 10^{-5}$ for all x in the interval $0 \leq x \leq 1$.

Q 4. (a) Find the radius of convergence and the interval of convergence of the following power series:

$$\sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{\sqrt{n+1} \cdot 4^n}.$$

(b) Let $f(x) = \sum_{n=0}^{\infty} \frac{(x+1)^{2n}}{\sqrt{n+1} \cdot 4^n}$. Find $f^{(126)}(-1)$ and $f^{(127)}(-1)$.

Q 5.

(a) Prove that $\displaystyle\lim_{(x,y)\to(1,0)} \frac{(x-1)y^2}{(x-1)^2+y^2} = 0.$

(b) Prove that $\displaystyle\lim_{(x,y)\to(1,0)} \frac{(x-1)y}{(x-1)^2+y^2}$ does not exist.