Date: 26 July 2010 Time: 17:00-19:30

NAME SURNAME:..

STUDENT NO:..

MATH 114 FINAL EXAM

IMPORTANT

1. This exam consists of 6 questions of different weights.
2. Each question is on a separate sheet. Please read the questions carefully and write your answers under the corresponding questions. Be neat.
3. Show all your work. Correct answers without sufficient explanation might not get full credit.
4. Calculators are not allowed.
5. Mobile telephones must be shut off.

Please do not write anything below this line.

1	2	3	4	5	6	TOTAL
17	17	17	15	15	19	100

The following formulas are given without any explanation. You may use them with your own interpretation and responsibility.

$$\text{Green's Theorem: } \oint_C M\,dx + N\,dy = \iint_D \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\,dA.$$

$$\text{Divergence Theorem: } \iiint_D \text{div}\mathbf{F}\,dV = \iint_S \mathbf{F} \bullet \hat{\mathbf{N}}\,dS.$$

$$\text{Stokes' Theorem: } \oint_C \mathbf{F} \bullet d\mathbf{r} = \iint_C \text{curl}\mathbf{F} \bullet \hat{\mathbf{N}}\,dS.$$

Q 1. (17 points) Given that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, \quad -1 < x < 1,$$

(a) find the Maclaurin series of $f(x) = \arctan x$ valid for $-1 < x < 1$. Which theorem(s) did you use?

(b) Show that the series that you found in part (a) converges also at the points $x = 1$ and $x = -1$ to $\arctan 1$ and $\arctan(-1)$? Which theorem(s) did you use?

Q 2. (17 points) Let D be the "rectangle-like" region in the first quadrant of the xy-plane, whose two sides are the lines $y = \sqrt{3}x$, $y = \frac{1}{\sqrt{3}}x$, and the other two sides are the circles $x^2 + y^2 = 1$, $x^2 + y^2 = 4$. Let $\mathcal{C}$ be the boundary of D, and assume $\mathcal{C}$ is oriented in the counterclockwise direction. Evaluate the following line integral

$$\oint_{\mathcal{C}} (3x^2y^2 - y^3)\,dx + (2x^3y + x^3)\,dy$$

Q 3. (17 points) Let

$$I = \int_{-\sqrt{3}}^{\sqrt{3}} dx \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} dy \int_{x^2+y^2}^{\sqrt{12-x^2-y^2}} z\, dz.$$

Write I as equivalent iterated triple integrals in
(a) cylindrical coordinate,
(b) spherical coordinates.
Do not evaluate the integrals.

Q 4. (15 points) Let $\mathcal{S}$ be the surface cut from the parabolic cylinder $z = 9 - x^2$ by the planes $z = 0$, $y = 0$, $y = 1$, $x = 0$. Evaluate

$$\iint_{\mathcal{S}} \sqrt{9 - z}\, dS.$$

Q 5. (15 points) Let $\mathcal{S}$ be the top of the paraboloid $z = 6 - x^2 - y^2$ cut by the plane $z = 2$, and assume $\mathcal{S}$ is oriented upward (that is away from the origin). Let

$$\mathbf{F} = -yz\mathbf{i} + xz\mathbf{j} + e^y \sin(x^3)\mathbf{k}.$$

By using a theorem, evaluate

$$\iint_{\mathcal{S}} \text{curl}\mathbf{F} \bullet \hat{\mathbf{N}}\, dS.$$

Which theorem did you use?

Q 6. (19 points) A function $f : \mathbb{R}^3 \to \mathbb{R}$ has continuous second order partial derivatives and is such that

- $f(x, y, z) \neq 0$ for all (x, y, z),
- $|\nabla f|^2 = 2f$,
- $\operatorname{div}(f\, \nabla f) = 5f$.

Evaluate $I = \displaystyle\iint_{\mathcal{S}} \nabla f \bullet \hat{\mathbf{N}}\, dS$, where $\mathcal{S}$ is the boundary of the solid cylinder $x^2 + y^2 \leq 4,\ 0 \leq z \leq 5$, and $\hat{\mathbf{N}}$ is the outer unit normal vector to $\mathcal{S}$.