Related topics
Bragg reflection, Debye-Scherrer method, lattice planes, graphite structure, material waves, de Broglie equation.

Principle and task
Fast electrons are diffracted from a polycrystalline layer of graphite: interference rings appear on a fluorescent screen. The interplanar spacing in graphite is determined from the diameter of the rings and the accelerating voltage.

Equipment
- Electron diff. tube a. mounting: 06721.00 1
- High voltage supply unit, 0-10 kV: 13670.93 1
- High-value resistor, 10 MΩ: 07160.00 1
- Connecting cord, 50 KV, 500 mm: 07366.00 1
- Power supply, 0...600 VDC: 13672.93 1
- Vernier caliper, plastic: 03011.00 1
- Connecting cord, 250 mm, red: 07360.01 2
- Connecting cord, 250 mm, blue: 07360.04 2
- Connecting cord, 750 mm, red: 07362.01 2
- Connecting cord, 750 mm, yellow: 07362.02 1
- Connecting cord, 750 mm, blue: 07362.04 1
- Connecting cord, 750 mm, black: 07362.05 2

Problems
1. To measure the diameter of the two smallest diffraction rings at different anode voltages.
2. To calculate the wavelength of the electrons from the anode voltages.
3. To determine the interplanar spacing of graphite from the relationship between the radius of the diffraction rings and the wavelength.

Set-up and procedure
Set up the experiment as shown in Fig. 1. Connect the sockets of the electron diffraction tube to the power supply as shown in Fig. 2. Connect the high voltage to the anode G3 through a 10 MΩ protective resistor.
where \( d \) is the spacing between the planes of the carbon atoms and \( \theta \) is the Bragg angle (angle between electron beam and lattice planes).

In polycrystalline graphite the bond between the individual layers (Fig. 3) is broken so that their orientation is random. The electron beam is therefore spread out in the form of a cone and produces interference rings on the fluorescent screen.

The Bragg angle \( \theta \) can be calculated from the radius of the interference ring but it should be remembered that the angle of deviation \( \alpha \) (Fig. 2) is twice as great:

\[
\alpha = 2 \theta.
\]

From Fig. 2 we read off

\[
\sin 2\alpha = \frac{r}{R}.
\]

where \( R = 65 \text{ mm} \), radius of the glass bulb.

Now, \( \sin 2\alpha = 2 \sin \alpha \cos \alpha \).

Theory and evaluation

To explain in the interference phenomenon, a wavelength \( \lambda \), which depends on momentum, is assigned to the electrons in accordance with the de Broglie equation:

\[
\lambda = \frac{h}{p}
\]

where \( h = 6.625 \cdot 10^{-34} \text{ Js} \), Planck’s constant.

The momentum can be calculated from the velocity \( v \) that the electrons acquire under acceleration voltage \( U_A \):

\[
\frac{1}{2} m v^2 = \frac{p^2}{2m} = e \cdot U_A
\]

The wavelength is thus

\[
\lambda = \frac{h}{\sqrt{2m} \cdot U_A}.
\]

where \( e = 1.602 \cdot 10^{-19} \text{ As} \) (the electron charge) and \( m = 9.109 \cdot 10^{-31} \text{ kg} \) (rest mass of electron).

At the voltages \( U_A \) used, the relativistic mass can be replaced by the rest mass with an error of only 0.5%.

The electron beam strikes a polycrystalline graphite film deposit on a copper grating and is reflected in accordance with the Bragg condition:

\[
2d \sin \theta = n \cdot \lambda, \ n = 1, 2, \ldots
\]

Set the Wehnelt voltage \( G1 \) and the voltages at grid 4 (G4) and G3 so that sharp, well-defined diffraction rings appear.

Read the anode voltage at the display of the HV power supply.

To determine the diameter of the diffraction rings, measure the inner and outer edge of the rings with the vernier caliper (in a darkened room) and take an average. Note that there is another faint ring immediately behind the second ring.
For small angles $\alpha$ ($\cos 10^\circ = 0.985$) can put

$$\sin 2\alpha = 2 \sin \alpha$$  \hspace{1cm} (6)

so that for small angles $\theta$ we obtain

$$\sin \alpha = \sin 2\theta \approx 2 \sin \theta$$  \hspace{1cm} (6a)

With this approximation we obtain

$$r = \frac{2R}{\lambda} \cdot n \cdot \lambda$$  \hspace{1cm} (7)

The two inner interference rings occur through reflection from the lattice planes of spacing $d_1$ and $d_2$ (Fig. 4), for $n = 1$ in (7).

The wavelength is calculated from the anode voltage in accordance with (3):

<table>
<thead>
<tr>
<th>$U_a$ (kV)</th>
<th>$\lambda$ (pm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.00</td>
<td>19.4</td>
</tr>
<tr>
<td>4.50</td>
<td>18.3</td>
</tr>
<tr>
<td>5.00</td>
<td>17.3</td>
</tr>
<tr>
<td>5.50</td>
<td>16.5</td>
</tr>
<tr>
<td>6.00</td>
<td>15.2</td>
</tr>
<tr>
<td>7.00</td>
<td>14.7</td>
</tr>
<tr>
<td>7.40</td>
<td>14.3</td>
</tr>
</tbody>
</table>

Applying the regression lines expressed by

$$Y = AX + B$$

Fig. 5: Radii of the first two interference rings as a function of the wavelength of the electrons.

Fig. 6: Interplanar spacing in graphite

$d_1 = 213$ pm, $d_2 = 123$ pm, $d_3 = 80.5$ pm, $d_4 = 59.1$ pm, $d_5 = 46.5$ pm.

to the measured values from Fig. 5 gives a slopes

$$A_1 = 0.62 (2) \cdot 10^9$$

$$A_2 = 1.03 (2) \cdot 10^9$$

and the lattice constants

$$d_1 = 211 \text{ pm}$$

$$d_2 = 126 \text{ pm}$$

in accordance with (7),

$$\frac{T_i}{\lambda} = A_i = \frac{2R}{\lambda_i}$$

and

$$d_i = \frac{2R}{\lambda_i}.$$

Notes

- The intensity of higher order interference rings is much lower than that of first order rings. Thus, for example, the second order ring of $d_1$ is difficult to identify and the expected fourth order ring of $d_4$ simply cannot be seen. The third order ring of $d_1$ is easy to see because graphite always has two lattice planes together, spaced apart by a distance of $d_1/3$. (Fig. 6)

In the sixth ring, the first order of ring of $d_4$ clearly coincides with the second order one of $d_2$. Radii (mm) calculated according to (4) for the interference rings to be expected when $U_a = 7$ kV:

<table>
<thead>
<tr>
<th>$n$</th>
<th>$d_1$ (pm)</th>
<th>$d_2$ (pm)</th>
<th>$d_3$ (pm)</th>
<th>$d_4$ (pm)</th>
<th>$d_5$ (pm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.9</td>
<td>15.4</td>
<td>23.2</td>
<td>31.0</td>
<td>38.5</td>
</tr>
<tr>
<td>2</td>
<td>15.7</td>
<td>29.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>26.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>34.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For small angles $\alpha$ ($\cos 10^\circ = 0.985$) can put

$$\sin 2\alpha = 2 \sin \alpha$$  \hspace{1cm} (6)

so that for small angles $\theta$ we obtain

$$\sin \alpha = \sin 2\theta \approx 2 \sin \theta$$  \hspace{1cm} (6a)
- The visibility of high order rings depends on the light intensity in the laboratory and the contrast of the ring system which can be influenced by the voltages applied to G1 and G4.

- The bright spot just in the center of the screen can damage the fluorescent layer of the tube. To avoid this reduce the light intensity after each reading as soon as possible.