Observation of Berry's Topological Phase by Use of an Optical Fiber

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We report the first experimental verification of Berry's topological phase. The key element in the experiment was a single-mode, helically wound optical fiber, inside which a photon of a given helicity could be adiabatically transported around a closed path in momentum space. The experiment confirmed at the classical level that the angle of rotation of linearly polarized light in this fiber gives a direct measure of Berry's phase. The topological nature of this effect was also verified, i.e., the rotation was found to be independent of deformations of fiber path if the solid angle of the path in momentum space stayed constant.

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Recently, Chiao and Wu[1] have pointed out some novel and observable quantum interference phenomena which arise from Berry's phase[2] for the photon. This phase, which is similar in many respects to the Aharonov-Bohm phase, has recently appeared theoretically in many fields of physics, from high-energy physics to low (e.g., from chiral anomalies in gauge field theories to a treatment of the Born-Oppenheimer approximation).[3] Hence it is important to look for Berry's phase experimentally. The optical effects predicted by Chiao and Wu allow such observations. The Bose nature of the photon permits optical manifestations of Berry's phase on a classical, macroscopic level, unlike the case of Fermi particles. Thus an intuitive understanding of this general phase factor emerges. One of their predictions is the appearance of an effective optical activity of a helically wound, single-mode optical fiber. They showed that the angle of rotation of linearly polarized light propagating down the fiber is a direct measure of Berry's phase. This optical activity does not come from a local elasto-optic effect caused by torsional stress,[4] but rather arises solely from the overall geometry of the path taken by the light, and hence is a global topological effect. Thus this effect is independent of the detailed material properties of the fiber.

In this Letter, we report an experimental study of the optical activity arising from Berry's phase in a single-mode fiber. To explore the topological nature of this effect, we compare the results from complex paths of nonuniform helices with those from simple uniform helices. We find good agreement between the measured rotation angles and those predicted by Berry's phase in all cases. These observations confirm the topological nature of this phase, which is one of its most significant properties. The rotation angle is found to be independent of the path of the fiber in configuration space as long as the solid angle subtended by the path in momentum space stays constant. In the special case of planar paths, no significant optical rotation is observed independent of the paths's complexity. Hence the light is able to distinguish between two and three spatial dimensions. This again confirms the topological nature of the effect.

Connection of earlier observations of optical activity in fibers,[5,6] with Berry's phase and its quantal, global topological properties went unnoticed. The observation of polarization rotation ascribed to geometrical effects was previously reported by Ross,[5] who used a single-mode fiber wound in a uniform helix, and by Varnham, Birch, and Payne,[6] who fabricated a fiber with a core wound into a uniform helix. Both papers studied the case where the helix was uniform, i.e., with a constant pitch. The observations were in good agreement with a classical analysis,[5,6] which treated the rotation of the plane of polarization locally at each point along the fiber for the case of a uniform helix by use of differential geometry.

The experimental setup is schematically shown in Fig. 1(a). A He-Ne laser and a pair of linear polarizers, one at the input, the other at the output end of the fiber, were used to measure the rotation of the plane of polarization in a 180-cm-long single-mode fiber. The fiber had a conventional step-index–type profile with a relative core-cladding index difference of 0.6%, and a core diameter of 2.6 μm. Its cladding index of refraction was 1.45 and its cladding diameter was 70 μm, which was coated with uv-curable acrylate of thickness $\sim 100\ \mu$m. The fiber was first inserted loosely in a Teflon sleeve in the form of a 175-cm-long tube, to minimize any torsional stress on the fiber during winding. The tube was wound helically with the output end of the fiber free to rotate. Thus care was taken not to introduce any torsional stress which might

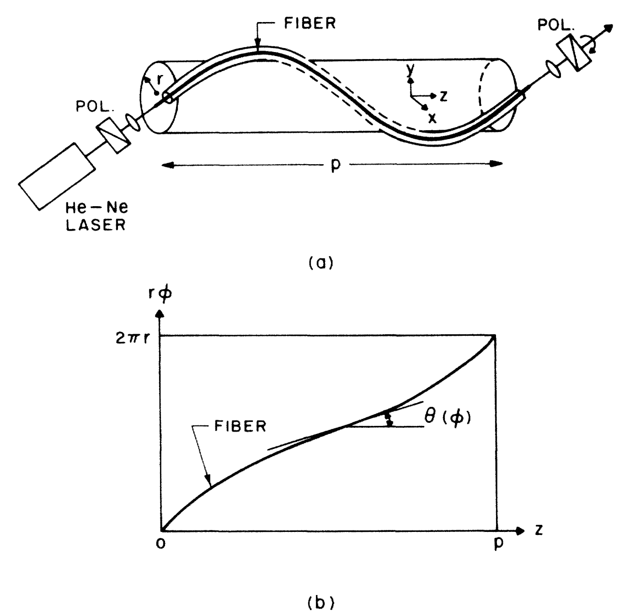

FIG. 1. (a) Experimental setup; (b) geometry used to calculate the solid angle in momentum space of a nonuniformly wound fiber on a cylinder.

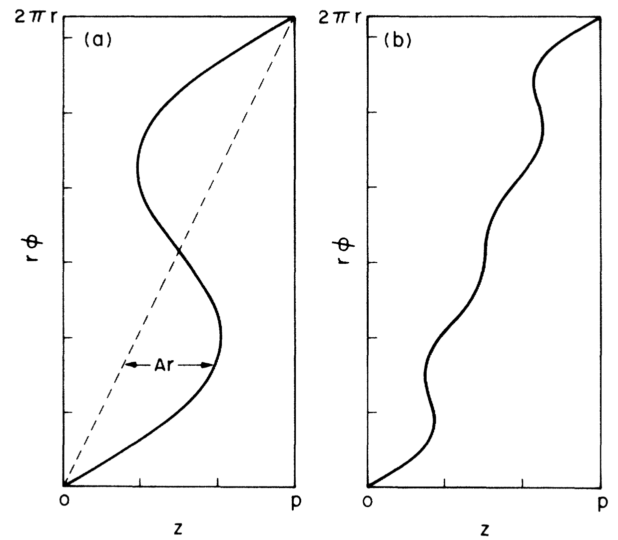

FIG. 2. (a) The solid line represents the path of the fiber on an unwrapped cylinder surface for nonuniform helices (squares in Figs. 3 and 4) with one harmonic of deformation [Eq. (5) with $A=1.2$], and the dashed line a uniform helix ($A=0$); (b) the path for a nonuniform helix (triangle in Figs. 3 and 4) with three harmonics of deformation [Eq. (6)].

result in a rotation of the plane of polarization due to the elasto-optic effect.[7] Also, we found that the fiber showed a negligibly small linear birefringence as long as the fiber was wound smoothly on a large enough diameter.[8] In order to form a closed path in momentum or **k** space,[1] the propagation directions of the input and output of the fiber were kept identical. In the first experiment, the fiber was wound into a uniform helix. The pitch angle of the helix θ, i.e., the angle between the local waveguide axis and the axis of the helix, was varied by attaching the Teflon sleeve along the outside perimeter of a spring, which was stretched from a tightly coiled configuration into a straight line. In this way, the pitch length p was varied, as was the radius r of the helix, but the fiber length $s=[p^2+(2\pi r)^2]^{1/2}$, i.e., the arc length of the helix, was kept constant. The range of p was from 30 to 175 cm. Hence the diameter of the helix ranged from 55 cm down to zero. By geometry, $\cos\theta = p/s$ [see Fig. 1(b)]. The solid angle in momentum space $\Omega(C)$ spanned by the fiber's closed path C in this space, in this case a circle, is $2\pi(1-\cos\theta)$. Berry's phase, $\gamma(C)=-\sigma\Omega(C)$,[1] for a single-turn uniform helix is therefore

$$\gamma(C)=-2\pi\sigma(1-p/s), \tag{1}$$

where $\sigma=\pm 1$ is the helicity quantum number of the photon. The quantum theory[1] predicts that $-\gamma_+(C)$, where $\gamma_+(C)$ is Berry's phase for $\sigma=+1$, is the angle Θ of rotation of linear polarization. The classical theory[5,6] predicts an angle of optical rotation in agreement with this quantum result.

In the second experiment, the fiber was wound onto a cylinder of a fixed radius to form a nonuniform helix. The procedure was first to wrap a piece of paper with a computer generated curve onto the bare cylinder. Then the Teflon sleeve with the fiber inside was laid on top of this curve. (To allow for variations in fiber path while using a fiber of fixed length, we left a straight section of fiber path at the output end, which had a variable length.) The solid angle in momentum space could then be calculated from the curve by unwrapping the paper onto a plane [see Figs. 1(b) and 2]. Let the horizontal axis of the paper, which was aligned with respect to the axis of the cylinder, be the z axis. Then the vertical axis represents $r\phi$, where r is the radius of the cylinder and $\phi=\tan^{-1}(y/x)$ is the azimuthal angle of a point on the curve with coordinates $(r\phi, z)$. The local pitch angle from Fig. 1(b),

$$\theta(\phi)=\tan^{-1}(r\, d\phi/dz), \tag{2}$$

characterizes the tangent to the curve followed by the fiber, and represents the angle between the local waveguide and the helix axes. In momentum space, $\theta(\phi+\pi/2)$ traces out a closed curve C corresponding to the fiber path on the surface of a sphere. The solid angle subtended by C with respect to the center of the sphere is given by

$$\Omega(C)=\int_0^{2\pi}[1-\cos\theta(\phi)]d\phi. \tag{3}$$

Berry's phase is then given by[1]

$$\gamma(C) = -\sigma\Omega(C). \quad (4)$$

One sees that Eq. (1) is a special case of Eqs. (3) and (4), when θ is a constant.

Figure 3 shows the measured rotation angle Θ versus the calculated solid angle $\Omega(C)$. The open circles represent the case of uniform helices, and the squares and the triangle represent nonuniform helices. The solid circles represent arbitrary planar curves formed by laying the fiber on a flat surface. The solid circle at $\Omega = 0$ corresponds to a snake-like path, and the one at $\Omega \approx 2\pi$ to a loop with a crossing. The squares represent helices with a single harmonic of deformation,

$$z/r = (p/2\pi r)\phi + A \sin\phi, \quad (5)$$

where $p = 42.6$ cm and $r = 14.2$ cm, and A ranges from 0 to 1.5 in steps of 0.3 [see Fig. 2(a)]. The triangle represents a helix with three harmonics of deformation,

$$z/r = (p/2\pi r)\phi + A_1 \sin\phi + A_2 \sin 2\phi + A_3 \sin 3\phi, \quad (6)$$

where $A_1 = A_2 = A_3 = 0.2$ [see Fig. 2(b)].

By inspection of Fig. 3, one sees that in all cases the measured rotations agree with the calculated magnitude of Berry's phase $|\gamma_+(C)|$ [see Eq. (4)] indicated by the solid line. The sense of the rotation, when one looks into the output end of the fiber, was found to be clockwise (i.e., dextrorotatory) for a left-handed helix, in agreement with theoretical prediction.[1]

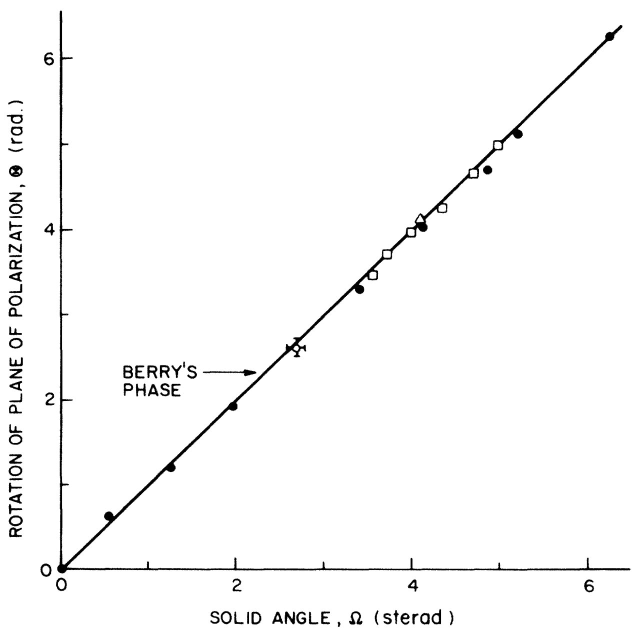

FIG. 3. Measured angle of rotation of linearly polarized light vs calculated solid angle in momentum space, Eq. (3). Open circles represent the data for uniform helices; squares and triangle represent nonuniform helices (see Fig. 2); solid circles represent arbitrary planar paths. The solid line is the theoretical prediction based on Berry's phase, Eq. (4).

The typical vertical error bar in Fig. 3 represents the dominant systematic error in this experiment, namely residual optical rotation due to torsional stress in the fiber. In separate auxiliary experiments, the optical rotation in a deliberately torsionally stressed fiber was measured, and also the residual strain, i.e., the twist of the fiber due to its rubbing against the walls of the Teflon sleeve, was measured microscopically near its free end. From these measurements, an estimate of size of the vertical error bar was determined. The typical horizontal error bar represents the uncertainty in the determination of the solid angle $\Omega(C)$ due to the fact that the fiber was free to roam within the 5-mm inner diameter of the Teflon tube. Random errors due to photon statistics were negligible compared with these systematic errors.

To check quantitatively the topological nature of the optical rotation, we replot the data in Fig. 3, as the slope $\Delta\Theta/\Delta\Omega$ of a line joining a datum point with the origin versus a deformation parameter D, onto Fig. 4. We define D as follows:

$$D = \left\{ \int_0^{2\pi} [1 - \cos\theta(\phi) - \Omega(C)/2\pi]^2 d\phi \right\}^{1/2} \Big/ \Omega(C). \quad (7)$$

Here D is a measure of the root mean square deviation of the fiber path from a uniform helix. By inspection of Fig. 4, one arrives at the conclusion that the specific optical rotation $\Delta\Theta/\Delta\Omega$ is in all cases independent of the deformation as quantified by the parameter D, and is therefore independent of geometry. This confirms the topological nature of Berry's phase. Since $\Delta\Theta/\Delta\Omega$ is a direct measure of σ,[1] one can view Fig. 4 as experimental evidence for the quantization of the "topological charge" of the system, which in this case is the he-

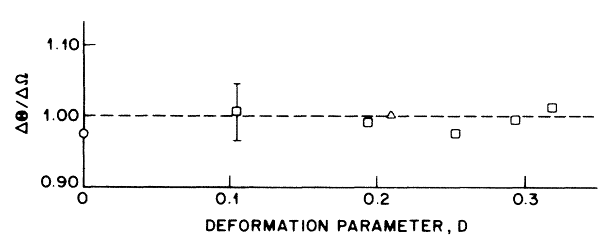

FIG. 4. The slopes $\Delta\Theta/\Delta\Omega$ of the points in Fig. 3 vs the deformation parameter D, Eq. (7), for nonuniform helices (squares and triangle). The open circle represents the average for all uniform helices. The dashed line represents the theoretical prediction.

licity of the photon, a relativistic quantum number.

The experiments reported here are essentially at the classical level, since we used an enormous number of photons in a single coherent state. Therefore at this point we can only say that we have verified the existence of Berry's phase and its topological properties at the classical level. These observations do support, however, the statement that these effects are "topological features of classical Maxwell theory which originate at the quantum level, but survive the correspondence-principle limit ($\hbar \to 0$) into the classical level."[1,9] It would be interesting to verify Berry's phase experimentally also at the quantum level, where fluctuations due to individual photons propagating inside the fiber appear. Then the truly quantum mechanical nature of this phase will become evident.

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Note added.—After we submitted this paper, another paper by G. Delacrétaz *et al.* [Phys. Rev. Lett. **56**, 2598 (1986)] was submitted and published, which independently verified the existence of Berry's phase experimentally in another context (i.e., the molecular system Na_3).

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[1]R. Y. Chiao and Y. S. Wu, preceding Letter [Phys. Rev. Lett. **57**, 933 (1986)]. Both in that paper and here, we adopt the sign conventions that $\Omega(C) > 0$ for a counterclockwise orientation of C with respect to the outward normal areal vector enclosed by C, by the right-hand rule in **k**-space. Thus the helix winding number $N > 0$ for a right-handed helix. Also, $\Theta > 0$ for dextrorotatory optical rotations [see F. A. Jenkins and H. E. White, *Fundamentals of Optics* (McGraw-Hill, New York, 1957), p. 572].

[2]M. V. Berry, Proc. Roy. Soc. London Ser. A **392**, 45 (1984).

[3]See Refs. 8–15 in Ref. 1.

[4]R. Ulrich and A. Simon, Appl. Opt. **18**, 2241 (1979).

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[6]M. P. Varnham, R. D. Birch, and D. N. Payne, in *Proceedings of the Fifth International Conference on Integrated Optics and Optical Fiber Communication and the Eleventh European Conference on Optical Communications* (Istituto Internazionale delle Communicazioni, Genova, Italy, 1985), p. 135.

[7]The straight, unstressed fiber possessed, however, an intrinsic circular birefringence due to the fiber drawing process, which produced an optical rotation of 0.436 rad/m. This number was checked by cutting the fiber into 30-cm sections. The rotation angles reported in the rest of this paper were measured with respect to the output polarization of the straight fiber as the zero reference. Also, $\kappa = 0.301$ rad/m in Eq. (10) of Ref. 1.

[8]The magnitude of the linear birefringence $n_{\parallel} - n_{\perp}$ was measured to be 1×10^{-9} and 5×10^{-9}, respectively, for the cases where the 180-cm fiber was laid straight, and where it was curved with a radius of 30 cm. The elliptical polarization of the output light of the fiber, under conditions where there were nonzero optical rotations arising from Berry's phase, was measured to be less than 2%, when the input light was essentially completely linearly polarized.

[9]J. H. Hannay, J. Phys. A. **18**, 221 (1985), has also discussed the classical limit of Berry's phase, e.g., in the case of a symmetric top. The angle Θ here is analogous to the angle that he found.